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SAVITRIBAI PHULE PUNE UNIVERSITY - 2015 SYLLABUS

T.E. (Elec.) Semester - II

CONTROL SYSTEM - I

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M.E. (Electrical)

Formerly Lecturer in Department of Electronics Engg.,
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SYLLABUS
CONTROL SYSTEM - I (303147)

Credits	Examination Scheme (Marks)
Th. : 04	In-Sem : 30 Marks
Pr. : 01	End-Sem : 70 Marks
	Oral : 50 Marks

Unit 01 : General

Basic concepts of control system, classification of control systems. Types of control system: Feedback, tracking, regulator system, feed forward system. Transfer function, Pole and zero concept. Modeling and representation of control system-Basic concept. Mechanical, Electrical and equivalent system, Electromechanical. Block diagram Algebra, signal flow graph, Mason's gain formula. **(Chapter - 1)**

Unit 02 : Time Domain Analysis

Standard test signal - step, ramp, parabolic and impulse signal, type and order of control system, time response of first and second order systems to unit impulse, unit step input. steady state errors - static error coefficients. Time domain specifications of second order systems. Importance of dominant closed loop poles of higher order systems Derivation of time domain specifications for second-order under-damped system for unit step input. **(Chapter - 2)**

Unit 03 : Stability Analysis and Root Locus

Concept of stability- Absolute, Asymptotic, relative and marginal. Nature of system response for various locations of roots in S-plane of characteristics equation. Routh's-Hurwithz criterion. Root Locus: Basic properties of root locus. Construction of root locus. Angle and magnitude condition for stable system. **(Chapter - 3)**

Unit 04 : Frequency Domain Analysis - I

Introduction, relation between time and frequency response for second order system. Frequency domain specifications, Polar Plot, Nyquist plot, stability analysis using Nyquist plot. **(Chapters - 4, 6)**

Unit 05 : Frequency Domain Analysis - II

Introduction to Bode plot, Asymptotic approximation: Sketching of Bode plot, stability, stability analysis using Bode plot. **(Chapter - 5)**

Unit 06 : PID Controllers

Basic concept of P, PI, PID controller, Design specifications in time domain and frequency domain. Design of PID controller by Root-Locus. Tuning of PID controllers. Ziegler-Nichol Method.

Control System Components : Working principle and transfer function of Lag network, lead network, potentiometer, AC and DC servo motors. Working principle of synchros. **(Chapter - 7)**

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IMPORTANT FORMULAE

Unit 01 : Mason's Gain Formula

$$\frac{C(s)}{R(s)} = \frac{\Sigma T_k \Delta_k}{\Delta}$$

Unit 02 : Time Domain Analysis

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \text{ where } E(s) = \frac{R(s)}{1 + G(s)H(s)}$$

Static error coefficient	Corresponding S.S. error e_{ss}
$K_p = \lim_{s \to 0} G(s)H(s)$	$\frac{A}{1+K_p}$
$K_v = \lim_{s \to 0} s\,G(s)H(s)$	$\frac{A}{K_v}$
$K_a = \lim_{s \to 0} s^2\,G(s)H(s)$	$\frac{A}{K_a}$

Type of system	Error coefficients			Error e_{ss} for input		
	K_p	K_v	K_a	Step input	Ramp input	Parabolic input
0	K	0	0	$\frac{A}{1+K}$	∞	∞
1	∞	K	0	0	$\frac{A}{K}$	∞
2	∞	∞	K	0	0	$\frac{A}{K}$

- $\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$ ... Standard 2nd order system

- The output of underdamped second order system to a unit step input is,

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{where}$$

$$\omega_d = \omega_n \sqrt{1-\xi^2} \quad \text{and} \quad \theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi^2}}{\xi}\right\} \text{radians}$$

- $T_d = \frac{1+0.7\xi}{\omega_n}$, $T_r = \frac{\pi-\theta}{\omega_d}$ sec, $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$ sec

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100, \quad T_s = \frac{4}{\xi\omega_n}, \quad T = \frac{1}{\xi\omega_n}$$

Unit 03 : Stability Analysis and Root Locus

Angles of asymptotes are,

$$\theta = \frac{(2\theta+1)\,180^\circ}{P-Z} \qquad \text{where } \theta = 0, 1, 2 \ldots\ldots (P-Z-1)$$

Centroid is given by,

$$\sigma = \frac{\sum \text{Real parts of poles of } G(s)H(s) - \sum \text{Real parts of zeros of } G(s)H(s)}{P-Z}$$

Breakaway points are the roots of $\frac{dK}{ds} = 0$.

Angles of departure and angles of arrivals at complex conjugate poles and zeros are,

$$\phi_d = 180^\circ - \phi \qquad \text{where } \phi = \sum \phi_P - \sum \phi_Z$$

$$\phi_a = 180^\circ + \phi \qquad \text{where } \phi = \sum \phi_P - \sum \phi_Z$$

Unit 04 : Frequency Domain Analysis - I

Resonant frequency, $\omega_r = \omega_n \sqrt{1-2\xi^2}$

Resonant peak, $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Unit 05 : Frequency Domain Analysis - II

$\text{G.M.} = -20 \operatorname{Log}_{10} |G(j\omega)H(j\omega)|_{\omega = \omega_{pc}}$

$\text{P.M.} = 180° + \angle G(j\omega)H(j\omega)|_{\omega = \omega_{gc}}$

$\omega_{gc} < \omega_{pc}$, G.M. and P.M. are positive, system is stable.

$\omega_{gc} > \omega_{pc}$, G.M. and P.M. are negative, system is unstable.

$\omega_{gc} = \omega_{pc}$, G.M. = P.M. = 0, system is marginally stable.

UNIT - 1

1 General

1.1 : Basic Concepts of Control Systems

Important Points to Remember

1. **Control system :** A control system is an arrangement of different physical elements connected in such a manner so as to regulate, direct or command itself or some other system.
2. **Plant :** The portion of a system which is to be controlled or regulated is called the **plant or the process.**
3. **Controller :** The element of the system itself or external to the system which controls the plant or the process is called **controller.**
4. **Input :** It is an applied signal or an excitation signal applied to a control system from an external energy source in order to produce a specified output.
5. **Output :** It is the particular signal of interest or an the actual response obtained from a control system when input is applied to it.

- The input variable is generally called as **reference input** denoted as r(t) while the output variable is called as **controlled output** denoted as c(t).

1.2 : Classification of Control System

Q.1 Explain how control systems are classified ?

☞ [SPPU : May-01, 03, 05, 08, Dec.-06, 10, 12, Marks 7]

Ans. : The control systems are broadly classified as,

i) Time varying and time invariant systems : Time varying contro systems are those in which parameters of the systems are varyin with time. The input and output may or may not be the function of time. For example, space vehicle whose mass decreases wit time, as it leaves earth. Time invariant systems are those in whic the parameters of system are independent of time, which are no varying with time and are constants. The example is the variou electrical networks consisting of resistance, inductances an capacitances.

ii) Linear and nonlinear systems : A systems is said to be linea it output varies linearily with input and it obeys the law o superposition. A simple resistive network is example of linea system. A system is said to be nonlinear if output is not linea function of input, it does not obey the law of superposition and it equations are nonlinear. A circuit consisting of diode as its elemen is nonlinear in nature.

iii) Continuous and sample data systems : In a continuous tim control system all system variables are the functions of continuous time variable 't'. The speed control of a d.c. moto using a tachogenerator feedback is an example of continuous dat system. In sample data systems one or more system variables ar known only at certain discrete intervals of time. They are no continuously dependent on the time. Microprocessor or compute based systems are the examples of sample data or digital systems.

iv) Deterministic and Stochastic Control Systems : A contro system is said to be deterministic when its response to input a well as behaviour to external disturbances is predictable an repeatable. If such response is unpredictable, system is said to b stochastic in nature.

v) Lumped Parameter and Distributed Parameter Contro Systems : Control systems using the parameters which ar physically separable and can be shown to be located at

i) Time varying and time invariant systems : Time varying control systems are those in which parameters of the systems are varying with time. The input and output may or may not be the functions of time. For example, space vehicle whose mass decreases with time, as it leaves earth. Time invariant systems are those in which the parameters of system are independent of time, which are not varying with time and are constants. The example is the various electrical networks consisting of resistance, inductances and capacitances.

ii) Linear and nonlinear systems : A systems is said to be linear it output varies linearily with input and it obeys the law of superposition. A simple resistive network is example of linear system. A system is said to be nonlinear if output is not linear function of input, it does not obey the law of superposition and its equations are nonlinear. A circuit consisting of diode as its element is nonlinear in nature.

iii) Continuous and sample data systems : In a continuous time control system all system variables are the functions of a continuous time variable 't'. The speed control of a d.c. motor using a tachogenerator feedback is an example of continuous data system. In sample data systems one or more system variables are known only at certain discrete intervals of time. They are not continuously dependent on the time. Microprocessor or computer based systems are the examples of sample data or digital systems.

iv) Deterministic and Stochastic Control Systems : A control system is said to be deterministic when its response to input as well as behaviour to external disturbances is predictable and repeatable. If such response is unpredictable, system is said to be stochastic in nature.

v) Lumped Parameter and Distributed Parameter Control Systems : Control systems using the parameters which are physically separable and can be shown to be located at a particular point while representing the system are lumped parameter systems. For example electrical networks with different parameters as resistance, inductance. etc. Control systems in which parameters **can not be physically separated** and hence can not be represented at a particular place are called distributed systems. The transmission line is an example as all its parameters like resistance, inductance etc. are distributed and can not be shown to be located at one place.

vi) Open and Closed loop Systems : A system in which output is function of input but input is not dependent on output is called open loop system. Automatic washing machine, traffic lights etc are open loop systems. A system in which the controlling action or input is somehow dependent on the output or changes in output is called closed loop system. Closed loop system uses the feedback property. The input is also dependent on output in such systems. Human being, postition control system, speed control system, missile systems etc. are examples of closed loop systems.

1.3 : Open Loop Control System

Q.2 Explain the open loop system with real time examples.

☞ **[SPPU : May-05,12, Dec.-02,03,07,09,11,13,14,16, Marks 3]**

Ans. : • A system in which output is dependent on input but controlling action or input is totally independent of the output or changes in output of the system, is called an Open Loop System.

- In a broad manner it can be represented as in Fig. Q.2.1.

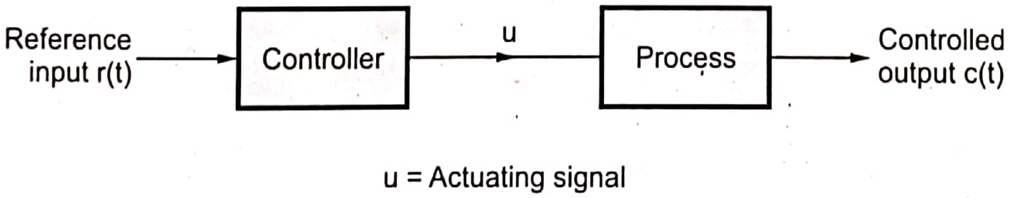


Fig. Q.2.1 Open loop control system

- Reference input [r(t)] is applied to the controller which generates the actuating signal (u) required to control the process which is to be controlled. Process is giving out the necessary desired controlled output c(t). The output has no effect on the input.

Real time examples : 1. An electric switch, 2. Automatic washing machine, 3. Automatic toaster system 4. Traffic light controller 5. Lawn watering system.

1.4 : Closed Loop Control System

Q.3 Define feedback. ☞ [SPPU : Dec.-15, Marks 2]

Ans. : Feedback is property of the system by which it permits the output to be compared with the reference input to generate the error signal, based on which the appropriate controlling action can be decieded.

Q.4 Explain the closed loop system with real time examples.
☞ [SPPU : May-08, 10, 12, Dec.-02, 03, 11, 13, 14, 16, Marks 3]

Ans. : • A system in which the controlling action or input is somehow dependent on the output or changes in output is called closed loop system. For this purpose such system uses the feedback property.

- In such system, output or part of the output is fedback to the input for comparison with the reference input applied to it.
- Closed loop system can be represented as shown in the Fig. Q.4.1.

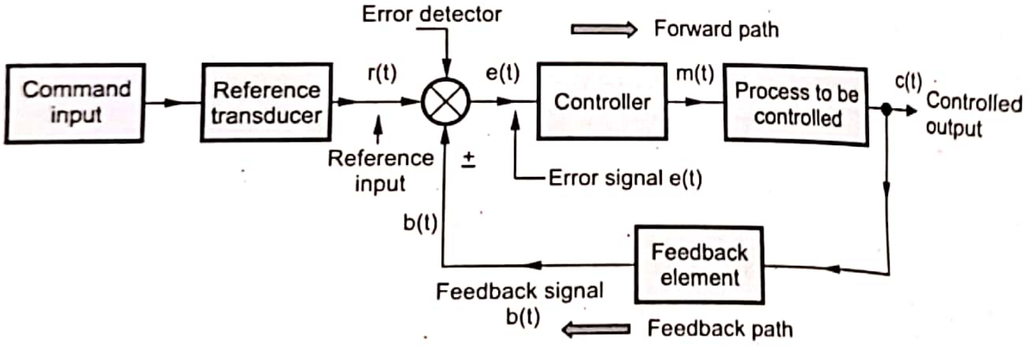

Error detector
Forward path
Command input
Reference transducer
r(t)
e(t)
Controller
m(t)
Process to be controlled
c(t)
Controlled output
Reference input
±
Error signal e(t)
b(t)
Feedback element
Feedback signal b(t)
Feedback path

Fig. Q.4.1 Representation of closed loop control system

- The main excitation to the system is called its **command input**
- The command input as it is may not be suitable for the comparison purpose hence it is given to the reference transducer to generate **reference input r(t).**
- The part of output, which is to be decided by feedback element is fed back to the reference input. The signal which is output of feedback element is called **'feedback signal'** b(t).
- It is then compared with the reference input using an error detector, giving error signal e(t) = r(t) ± b(t). This is called **actuating signal** for the process.
- This error signal is then modified by controller and decides the proportional **manipulated signal** m(t) for the process to be controlled.
- The process to be controlled is called a **plant.**
- This manipulation is such that error will approach zero. This signal then actuates the actual system and produces an output. As output is controlled one, hence called **controlled output** c(t).

Real time examples : 1. Human being reaching for an object, 2. Closed loop temperature control system, 3. Ship stabilization system, 4. D.C. motor speed control system 5. Auto pilot system for air crafts, 6. Missile launching system, 7. Sun seeker solar system.

1.5 : Comparison of Open and Closed Loop Control Systems

Q.5 Compare open loop and closed loop systems.

☞ **[SPPU : April-15,16,17 (In Sem), May-16, Dec.-16,17, Marks 4]**

Ans. :

Sr. No.	Open Loop	Closed Loop
1.	Any change in output has no effect on the input i.e. feedback does not exists.	Changes in output, affects the input which is possible by use of feedback.

2.	Output measurement is not required for operation of system.	Output measurement is necessary.
3.	Feedback element is absent.	Feedback element is present.
4.	Error detector is absent.	Error detector is necessary.
5.	It is inaccurate and unreliable.	Highly accurate and reliable.
6.	Highly sensitive to the disturbances.	Less sensitive to the disturbances.
7.	Highly sensitive to the environmental changes.	Less sensitive to the environmental changes.
8.	Bandwidth is small.	Bandwidth is large.
9.	Simple to construct and cheap.	Complicated to design and hence costly.
10.	Generally are stable in nature.	Stability is the major consideration while designing.
11.	Highly affected by nonlinearities.	Reduced effect of nonlinearities.
12.	The examples are electric switch, traffic light controller, automatic washing machine, automatic toaster, water sprinkler etc.	The examples are human being, temperature control, speed control, ship stabilization, auto pilots for aircrafts, inertial guidance system, automatic electric iron, sunseeker solar system, water level controllers etc.

1.6 : Tracking and Regulator Control Systems

Q.6 Define tracking and regulator control systems.

☞ [SPPU : April-15 (In Sem), Dec.-15, Marks 2]

Ans. : i) **Tracking control system** : A system in which the reference input is time variant and output follows this input as closely as possible is called tracking control system. The plant or process is required to track some prescribed function in this system. The radar antenna system used to track an aircraft is an example of tracking control system.

ii) Regulator control system : A regulator control system is the one in which the controlled variable is maintained at a constant value though there is an external load on the plant. Though the reference input is adjustable, it is maintained constant in such system. Thus main objective of a regulator control system is to maintain the output at the desired value in the presence of disturbances. The speed control system, regulating voltage of an alternator are the examples of regulator control system.

1.7 : Feedforward System

Q.7 Define and explain feedforward system.

☞ [SPPU : April-15 (In Sem), Dec.-15, Marks 2]

Ans. : If the distrubance is measurable, then the signal can be added to the controller input to modify the actuating signal. Thus a corrective action can be intiated without waiting for its effect to show up in the output. Such disturbances can be compensated before they affect the output. Such systems in which corrective action is taken before the disturbances affect the output are called feedfoward systems.

- The block diagram of feedfoward system is shown in the Fig. Q.7.1.

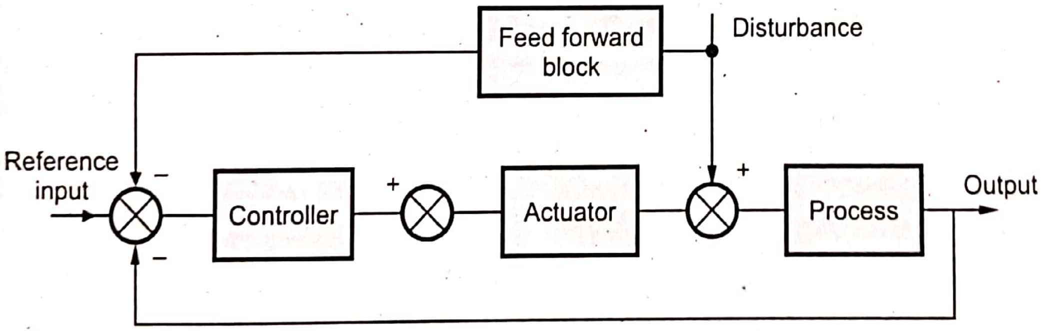


Fig. Q.7.1

Q.8 Distinguish between feedback and feedforward system.

☞ [SPPU : April-17, 18 (In Sem), Dec.-16, Marks 3]

Ans. :

Sr. No.	Feedback system	Feedforward system
1.	Corrective action starts only after disturbance affects the output.	Corrective action starts before disturbance affects the output.
2.	Feedback compensation is closed loop in nature.	Feedforward compensation is open loop in nature.
3.	It needs the measurement of target parameters.	It does not need the measurement of target parameters.
4.	It is reactive in nature.	It is proactive in nature.
5.	It can compensate for unmeasurable disturbances.	It can not compensate for unmeasurable disturbances.
6.	Variables are adjusted depending on errors.	Variables are adjusted based on prior knowledge and predictions.
7.	Example is use of roll sensor in ship stabilization as feedback element.	Use of flowmeter as feedforward block in a temperature control system.

1.8 : Transfer Function

Q.9 Define transfer function of a system.

☞ [SPPU : April-17 (In Sem), May-15, Dec.-15, Marks 2]

Ans. : Mathematically transfer function is defined as the ratio of Laplace transform of output (response) of the system to the Laplace transform of input (excitation or driving function), under the assumption that ***all initial conditions are zero.***

If T(s) is the transfer function of the system then,

$$\boxed{T(s) = \frac{\text{Laplace transform of output}}{\text{Laplace transform of input}} = \frac{C(s)}{R(s)}}$$

Important Points to Remember

Steps to determine transfer Function of a System

1) Draw the Laplace domain (s-domain) network of the given network. While ontaining s-domain network, replace R by R, L by sL, C by $\frac{1}{sC}$ and all time domain variables by Laplace domain variables such as v(t) by V(s), i(t) by I(s) and so on.
2) Identify system input and output variables in Laplace domain.
3) Eliminating other introduced variables, get the resultant equation in terms of input and output variables, in Laplace domain.
4) Take the ratio of Laplace transform of output variable to Laplace transform of input variable to get the transfer function model of the system interms of elements of the system.

Q.10 Derive transfer function for following electrical system.

☞ [SPPU : April-16, (In Sem), Marks 6]

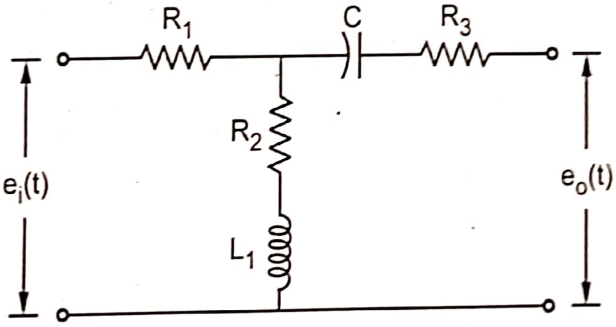


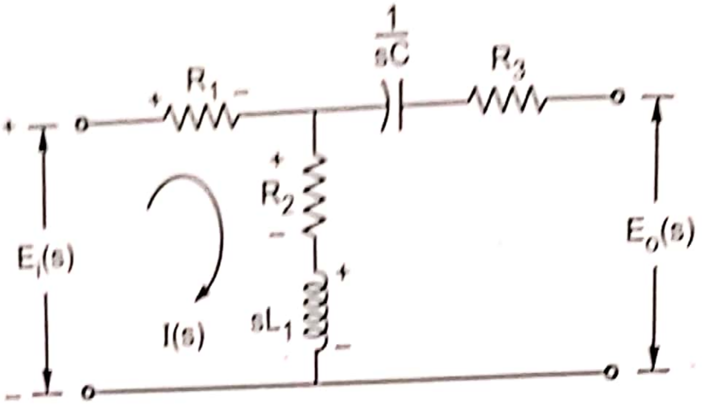
Fig. Q.10.1

Ans. : The s-domain network is shown in the Fig. Q.10.1(a). Applying KVL,

$$-I(s)R_1 - I(s)[R_2 + sL_1] + E_i(s) = 0$$

$$\therefore \qquad I(s) = \frac{E_i(s)}{R_1 + R_2 + sL_1}$$

No current flows through $\frac{1}{sC}$ and R_3.



$\frac{1}{sC}$
R_1
R_3
R_2
sL_1
$E_i(s)$
$E_o(s)$
I(s)

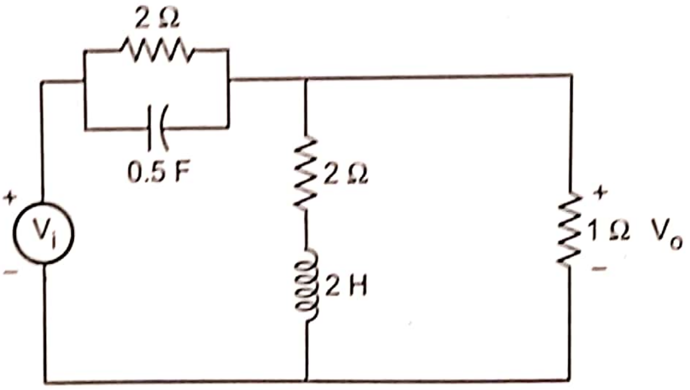
Fig. Q.10.1(a)

$$\therefore \quad E_o(s) = I(s) \times (R_2 + sL_1) = \frac{E_i(s)(R_2 + sL_1)}{R_1 + R_2 + sL_1}$$

$$\therefore \quad \text{T.F.} = \frac{E_o(s)}{E_i(s)} = \boxed{\frac{R_2 + sL_1}{R_1 + R_2 + sL_1}}$$

Q.11 Determine transfer function of following electrical network.

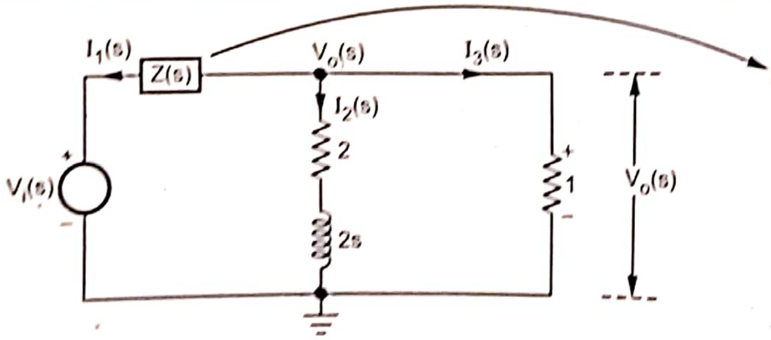
[SPPU : May-17, Marks 7]



2 Ω
0.5 F
2 Ω
2 H
V_i
1 Ω V_o

Fig. Q.11.1

Ans. : The s - domain network is shown in the Fig. Q.11.1(a).



$I_1(s)$
Z(s)
$V_o(s)$
$I_3(s)$
$I_2(s)$
2
2s
1
$V_i(s)$
$V_o(s)$

$$Z(s) = \frac{1}{0.5s} \parallel 2 = \frac{2 \times \frac{2}{s}}{2 + \frac{2}{s}} = \frac{2}{s+1}$$

Fig. Q.11.1 (a)

Using node analysis, the node voltage is $V_o(s)$ and three branch currents are $I_1(s)$, $I_2(s)$, $I_3(s)$.

Using KCL, $I_1(s) + I_2(s) + I_3(s) = 0$

$$I_1(s) = \frac{V_o(s) - V_i(s)}{Z(s)},\ I_2(s) = \frac{V_o(s)}{2+2s},\ I_3(s) = \frac{V_o(s)}{1}$$

$$\therefore \quad \frac{V_o(s) - V_i(s)}{Z(s)} + \frac{V_o(s)}{2(s+1)} + V_o(s) = 0$$

$$\therefore \quad V_o(s)\left[\frac{1}{Z(s)} + \frac{1}{2(s+1)} + 1\right] = \frac{V_i(s)}{Z(s)}$$

$$\therefore \quad \frac{V_o(s)}{V_i(s)} = \frac{2(s+1)Z(s)}{Z(s)[2(s+1) + Z(s) + 2Z(s)(s+1)]}$$

Using the value of Z(s) and solving,

$$\text{T.F.} = \frac{V_o(s)}{V_i(s)} = \boxed{\frac{s^2 + 2s + 1}{s^2 + 4s + 4}}$$

Q.12 Find the transfer function $V_o(s)/V_i(s)$ of the circuit shown in Fig. Q.12.1, if L = 1 H and C = 1 F ☞ **[SPPU : May-18, Marks 7]**

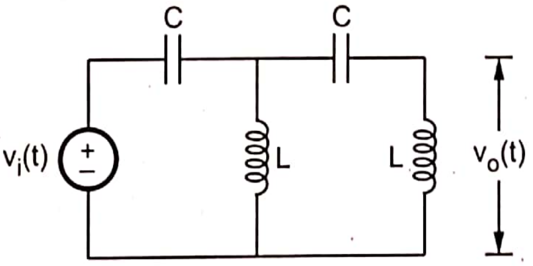


Fig. Q.12.1

Ans. : The s - domain network is shown in the Fig. Q.12.1(a).

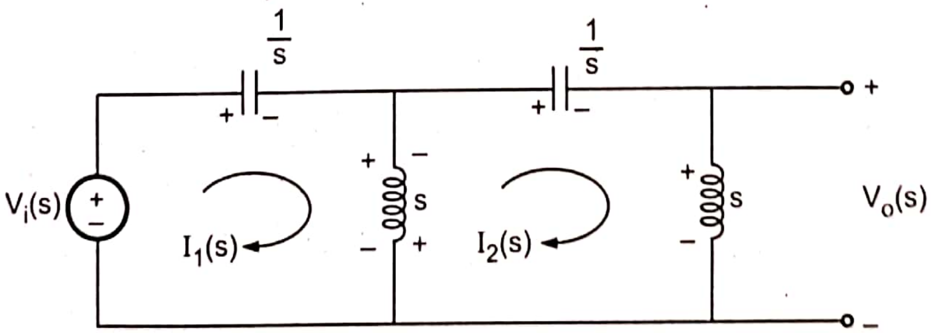


Fig. Q.12.1 (a)

Applying KVL to the two loops,

$$-\frac{1}{s}I_1(s)-sI_1(s)+sI_2(s)+V_i(s) = 0$$

i.e.
$$I_1(s)\left[\frac{s^2+1}{s}\right]-I_2(s)[s] = V_i(s) \qquad \ldots(1)$$

$$-\frac{1}{s}I_2(s)-sI_2(s)-sI_2(s)+sI_1(s) = 0$$

i.e.
$$sI_1(s)-I_2(s)\left[\frac{2s^2+1}{s}\right] = 0 \qquad \ldots(2)$$

$$D = \begin{vmatrix} \frac{s^2+1}{s} & -s \\ s & -\left[\frac{2s^2+1}{s}\right] \end{vmatrix} = \frac{-(s^4+3s^2+1)}{s^2}$$

$$D_2 = \begin{vmatrix} \frac{s^2+1}{s} & V_i(s) \\ s & 0 \end{vmatrix} = -sV_i(s)$$

$$\therefore \quad I_2(s) = \frac{D_2}{D} = \frac{s^3V_i(s)}{s^4+3s^2+1}$$

$$\therefore \quad V_o(s) = I_2(s)\times s = \frac{s^4V_i(s)}{s^4+3s^2+1}$$

$$\therefore \quad \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s^4}{s^4+3s^2+1}} \qquad \ldots \text{T.F.}$$

1.9 : Pole and Zero Concept

Q.13 Define and explain the following terms related to the transfer function of a system : i) Poles ii) Zeros iii) Characteristic equation iv) Pole-zero plot v) Order.

☞ [SPPU : April-17, (In Sem), Marks 4]

Ans. : As transfer function is a ratio of Laplace of output to Laplace of input and hence can be expressed as a ratio of polynomials in 's'.

- The numerator and denominator can be factorised to get the factorised form of the transfer function as,

$$\boxed{\text{T.F.} = \frac{P(s)}{Q(s)} = \frac{K(s-s_a)(s-s_b)\ldots\ldots(s-s_m)}{(s-s_1)(s-s_2)\ldots\ldots(s-s_n)} \quad \text{where } K = \text{System gain}}$$

i) Poles : The values of 's', which make the transfer function value infinite after substitution in the denominator of a T.F. are called 'Poles' of that T.F.

- The values $s_1, s_2, s_3 \ldots\ldots s_n$ in the above equation of T.F. are called poles of the T.F.

ii) Zeros : The values of 's' which make the value of transfer function zero after substituting in the numerator are called 'zeros' of that T.F.

- The values $s = s_a, s_b, \ldots\ldots, s_m$ in the above equation of T.F. are called zeros of the T.F.

iii) Characteristic equation : The equation obtained by equating denominator of a T.F. to zero, whose roots are the poles of that T.F. is called characteristic equation of that system.

- Thus Q(s) = 0 is the characteristic equation for the above T.F.

iv) Pole-Zero plot : Plot obtained by locating all poles and zeros of a T.F. in s-plane is called pole-zero plot of a system.

- The poles are indicated by 'cross' while zeros are indicated by 'small circles' in the pole-zero plot.

v) Order : The highest power of 's' present in the characteristic equation i.e. in the denominator polynomial of a closed loop transfer function of a system is called 'Order' of a system.

Q.14 The transfer function of a system is given by,

$$\mathbf{T(s) = \frac{K(s+2)}{s(s+4)(s^2+2s+2)}}$$

Determine : i) Poles ii) Zeros iii) Pole zero plot in s plane iv) Characteristic equation. ☞ [SPPU : May-15, Marks 6]

Ans. : The roots of s^2+2s+2 are $-1 \pm j$.

i) Poles : $s = 0, -4, -1 \pm j$

ii) Zeros : $s = -2$

iii) Pole-zero plot is shown in the Fig. Q.14.1.

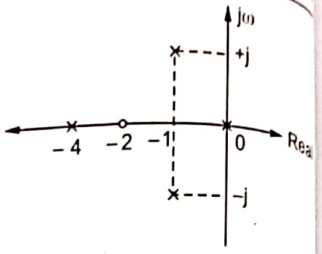


Fig. Q.14.1

iv) Characteristic equation :

$s(s+4)(s^2+2s+2) = 0$

i.e. $s^4 + 6s^3 + 10s^2 + 8s = 0$

1.10 : Modeling and Representation of Control Systems

Important Points to Remember

- The set of mathematical equations, describing the dynamic characteristics of a system is called mathematical model of the system. Obtaining the mathematical model by analyzing the system is called **mathematical modeling** of a system.
- In mechanical system the various elements corresponding to translational and rotational systems are,

Translational system	Mass (M)	Friction (B) or (D)	Spring (K)	Force	x(t)	v(t)	a(t)
Rotational system	Inertia (J)	Friction (B) or (D)	Spring (K)	Torque	θ(t)	ω (t)	α (t)

- $x(t)$ = Linear displacement, $\theta(t)$ = Angular displacement
- $v(t) = \left(\frac{dx}{dt}\right)$ = Velocity, $\omega(t) = \left(\frac{d\theta}{dt}\right)$ = Angular velocity

$a(t) = \left(\frac{d^2x}{dt^2}\right)$ = Acceleration, $\alpha(t) = \left(\frac{d^2\theta}{dt^2}\right)$ = Angular acceleration

- Remember that only spring and friction can cause change in displacement if connected between the two moving masses. The mass can not cause change in displacement hence can not exist between the nodes. The spring and friction with respect to fixed support can not cause change in displacement.

1.11 : Force-Voltage Analogy

Q.15 Explain the force-voltage analog. Mention the analogous elements based on force-voltage analogy.

☞ [SPPU : April-16, (In Sem), May-16,17, Dec.-17, 18, Marks 4]

Ans. : • Consider simple mechanical system as shown in the Fig. Q.15.1.

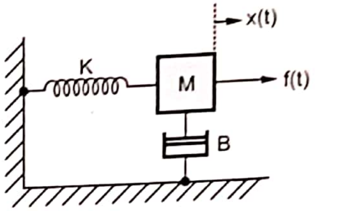


Fig. Q.15.1

- Due to the applied force, mass M will displace by an amount x(t) in the direction of the force f(t).
- The equilibrium equation is,

$$\therefore \; f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + K\,x(t)$$

Taking Laplace transform, $\boxed{F(s) = Ms^2X(s) + BsX(s) + KX(s)}$ ... (1)

- Consider electric network as shown in the Fig. Q.15.2.
- The equation according to Kirchhoff's voltage law can be written as,

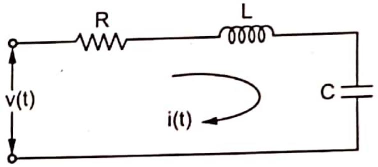


Fig. Q.15.2

$$v(t) = i(t)\,R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

- Taking Laplace,

$$V(s) = I(s)\,R + Ls\,I(s) + \frac{I(s)}{sC}$$

- But we cannot compare F(s) and V(s) unless we bring them into same form. For this we will use current as rate of flow of charge.

$$\therefore \quad i(t) = \frac{dq}{dt} \quad \text{i.e.} \quad I(s) = s\,Q(s) \quad \text{or} \quad Q(s) = \frac{I(s)}{s}$$

- Replacing in above equation,

$$\boxed{V(s) = L\,s^2Q(s) + Rs\,Q(s) + \frac{1}{C}Q(s)} \quad \text{... (2)}$$

i) Poles : $s = 0, -4, -1 \pm j$

ii) Zeros : $s = -2$

iii) Pole-zero plot is shown in the Fig. Q.14.1.

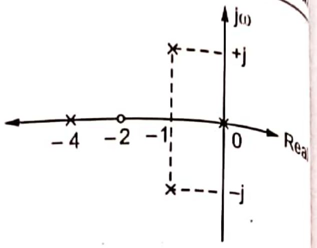


Fig. Q.14.1

iv) Characteristic equation : $s(s+4)(s^2+2s+2) = 0$

i.e. $s^4 + 6s^3 + 10s^2 + 8s = 0$

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Rotational system	Inertia (J)	Friction (B) or (D)	Spring (K)	Torque	θ(t)	ω (t)	α (t)

- $x(t)$ = Linear displacement, $\theta(t)$ = Angular displacement
- $v(t) = \left(\frac{dx}{dt}\right)$ = Velocity, $\omega(t) = \left(\frac{d\theta}{dt}\right)$ = Angular velocity

  $a(t) = \left(\frac{d^2x}{dt^2}\right)$ = Acceleration, $\alpha(t) = \left(\frac{d^2\theta}{dt^2}\right)$ = Angular acceleration
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Ans. : • Consider simple mechanical system as shown in the Fig. Q.15.1.

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- The equilibrium equation is,

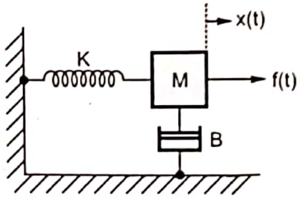

Fig. Q.15.1

$$\therefore \quad f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + Kx(t)$$

Taking Laplace transform, $\boxed{F(s) = Ms^2X(s) + BsX(s) + KX(s)}$ ... (1)

- Consider electric network as shown in the Fig. Q.15.2.
- The equation according to Kirchhoff's voltage law can be written as,

$$v(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

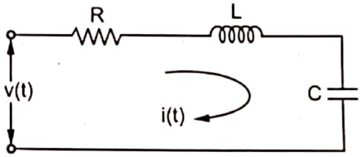

Fig. Q.15.2

- Taking Laplace,

$$V(s) = I(s)R + Ls\,I(s) + \frac{I(s)}{sC}$$

- But we cannot compare F(s) and V(s) unless we bring them into same form. For this we will use current as rate of flow of charge.

$$\therefore \qquad i(t) = \frac{dq}{dt} \quad \text{i.e.} \quad I(s) = s\,Q(s) \quad \text{or} \quad Q(s) = \frac{I(s)}{s}$$

- Replacing in above equation,

$$\boxed{V(s) = L\,s^2Q(s) + Rs\,Q(s) + \frac{1}{C}Q(s)} \quad \text{... (2)}$$

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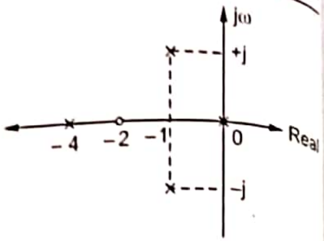

Fig. Q.14.1

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Rotational system	Inertia (J)	Friction (B) or (D)	Spring (K)	Torque	θ(t)	ω (t)	α (t)

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1.11 : Force-Voltage Analogy

Q.15 Explain the force-voltage analog. Mention the analogous elements based on force-voltage analogy.

☞ [SPPU : April-16, (In Sem), May-16,17, Dec.-17, 18, Marks 4]

Ans. : • Consider simple mechanical system as shown in the Fig. Q.15.1.

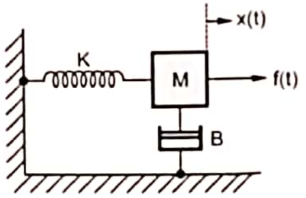


Fig. Q.15.1

- Due to the applied force, mass M will displace by an amount x(t) in the direction of the force f(t).
- The equilibrium equation is,

$$\therefore \quad f(t) = M\frac{d^2x(t)}{dt^2} + B\frac{dx(t)}{dt} + K\,x(t)$$

Taking Laplace transform, $\boxed{F(s) = Ms^2X(s) + BsX(s) + KX(s)}$ ... (1)

- Consider electric network as shown in the Fig. Q.15.2.

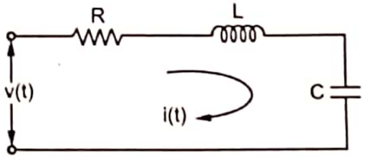


Fig. Q.15.2

- The equation according to Kirchhoff's voltage law can be written as,

$$v(t) = i(t)\,R + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt$$

- Taking Laplace,

$$V(s) = I(s)\,R + Ls\,I(s) + \frac{I(s)}{sC}$$

- But we cannot compare F(s) and V(s) unless we bring them into same form. For this we will use current as rate of flow of charge.

$$\therefore \quad i(t) = \frac{dq}{dt} \quad \text{i.e.} \quad I(s) = s\,Q(s) \quad \text{or} \quad Q(s) = \frac{I(s)}{s}$$

- Replacing in above equation,

$$\boxed{V(s) = L\,s^2Q(s) + Rs\,Q(s) + \frac{1}{C}Q(s)} \quad \text{... (2)}$$

- Comparing equations (1) and (2) various analogous quantitie based on F-V analogy are,

Translational	Rotational	Electrical
Force	Torque T	Voltage V
Mass M	Inertia J	Inductance L
Friction constant B	Tortional friction constant B	Resistance R
Spring constant K N/m	Tortional spring constant K Nm/rad	Reciprocal of capacitor 1/C
Displacement 'x'	θ	Charge q
Velocity $\dot{x} = \frac{dx}{dt}$	$\dot{\theta} = \frac{d\theta}{dt} = \omega$	Current $i = \frac{dq}{dt}$

Table Q.15.1 Tabular form of force-voltage analogy

1.12 : Force-Current Analogy

Q.16 Explain the force-current analog. Mention the analogous elements based on force-current analogy.

☞ [SPPU : May-16, Dec.-17, Marks 4]

Ans. : Consider the simple mechanical system shown in the Fig. Q.16.1 and its equlibrium equation is as obtained in the answe of Q.15 as,

$$\boxed{F(s) = Ms^2 X(s) + Bs\, X(s) + K\, X(s)} \quad ...(1$$

- Consider an electrical system based on node basis as shown in the Fig. Q.16.1.
- The equation according to Kirchhoff's current law for above system is,

$$I = I_L + I_R + I_C$$

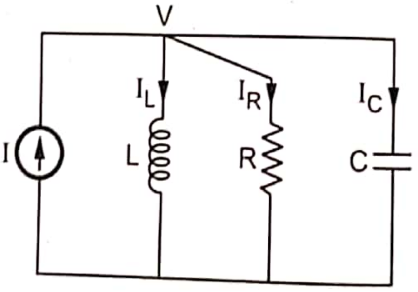


Fig. Q.16.1

- Let node voltage be V,

$$\therefore \quad I = \frac{1}{L}\int V dt + \frac{V}{R} + C\frac{dV}{dt}$$

- Taking Laplace transform,

$$I(s) = \frac{V(s)}{sL} + \frac{V(s)}{R} + sC\,V(s)$$

- But to get this equation in the similar form as that of F(s) we will use, $v(t) = \frac{d\,\phi}{dt}$ i.e. $V(s) = s\phi(s)$ i.e. $\phi(s) = \frac{V(s)}{s}$

  where ϕ = Flux

- Substituting in equation for I(s),

$$\boxed{I(s) = Cs^2\,\phi(s) + \frac{1}{R}s\,\phi(s) + \frac{1}{L}\phi(s)} \quad \text{... (2)}$$

- Comparing equations (1) and (2) various analogous quantities based on F-I analogy are,

Translational	Rotational	Electrical
F Force	T	Current I
M Mass	J	Capacitor C
B friction	B	Reciprocal of resistance, 1/R
K Spring	K	Reciprocal of inductance, 1/L
x displacement	θ	ϕ
$\dot{x}$ Velocity $= \frac{dx}{dt}$	$\dot{\theta} = \frac{d\theta}{dt} = \omega$	Voltage 'e' $= \frac{d\phi}{dt}$

Table Q.16.1 Tabular form of force-current analogy

1.13 : Equivalent Electrical Mechanical Systems

Important Points to Remember

Steps to draw equivalent mechanical system :

Step 1 : Due to applied force, identify the displacements in the given mechanical system.

Step 2 : Identify the elements which are under the influence of different displacements.

Step 3 : Represent each displacement by a separate node, using nodal Analysis.

Step 4 : Show all the elements in parallel under the respective nodes which are under the influence of respective displacements.

Step 5 : Elements causing same change in displacement will get connected in parallel in between the respective nodes.

Q.17 How to obtain electrical-mechanical analogous systems ?

Ans. :

Step 1 : Identify all the displacements due to the applied force.

Step 2 : Draw the equivalent mechanical system based on nod basis.

Step 3 : Write the equilibrium equations. At each node algebrai sum of all the forces acting at the node is zero.

Step 4 : In F-V analogy, use following replacements and rewrit equations,

$$F \rightarrow V, \quad M \rightarrow L, \quad B \rightarrow R, \quad K \rightarrow 1/C,$$

$$x \rightarrow q, \quad \dot{x} \rightarrow i \text{ (current)}, \; x \rightarrow \int i\,dt, \quad \ddot{x} \rightarrow \frac{d^2x}{dt^2} \rightarrow \frac{di}{dt}$$

Step 5 : Simulate the equations using loop method. Number o displacements equal to number of loop currents.

Step 6 : In F-I analogy, use following replacements and rewrit equations,

$$F \rightarrow I, \quad M \rightarrow C, \quad B \rightarrow 1/R, \quad K \rightarrow 1/L,$$

$$x \rightarrow \phi, \quad \dot{x} = v \text{ (voltage)}, \quad x \rightarrow \int v\,dt, \quad \ddot{x} \rightarrow \frac{d^2x}{dt^2} \rightarrow \frac{dv}{dt}$$

Step 7 : Simulate the equations using node basis for F-I analogus system.

Q.18 Draw mechanical equivalent network of a given system and also draw the electrical analogous circuit using F-V analogy.

[SPPU : Dec.-16, Marks 8]

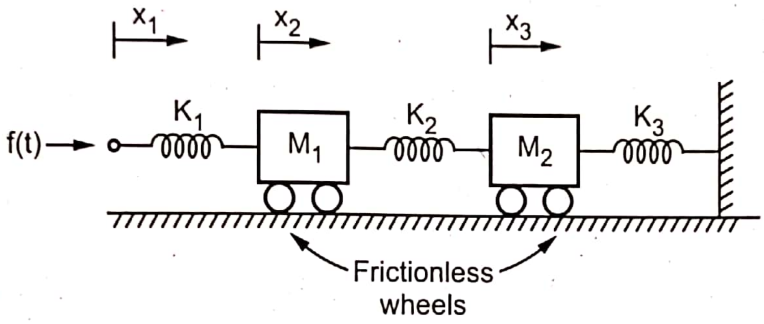


Fig. Q.18.1

Ans. : K_1 is under displacement $(x_1 - x_2)$. M_1 is under displacement x_2. K_2 is under the displacement $(x_2 - x_3)$. M_2 and K_3 are under the displacement x_3 alone. Hence equivalent mechanical network is shown in the Fig. Q.18.1 (a).

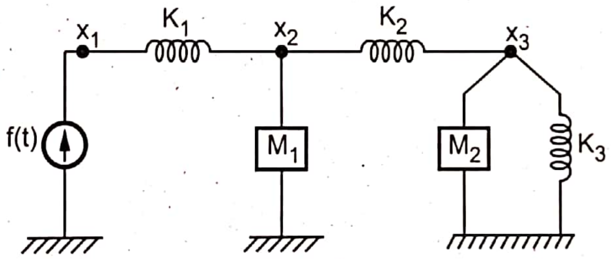


Fig. Q.18.1 (a)

The equations are,

$$f(t) = K_1(x_1 - x_2)$$

$$0 = K_1(x_2 - x_1) + K_2(x_2 - x_3) + M_1 \frac{d^2 x_2}{dt^2}$$

$$0 = K_2(x_3 - x_2) + M_2 \frac{d^2 x_3}{dt^2} + K_3 x_3$$

Using F - V analogy, $M \to L$, $B \to R$, $K \to \frac{1}{C}$, $x \to \int i\, dt$, $\frac{dx}{dt} \to i$, $\frac{d^2 x}{dt^2} \to \frac{di}{dt}$, $f(t) \to v(t)$

$$\therefore \quad v(t) = \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$0 = \frac{1}{C_1}\int (i_2 - i_1)\,dt + \frac{1}{C_2}\int (i_2 - i_3)\,dt + L_1 \frac{di_2}{dt}$$

$$0 = \frac{1}{C_2}\int (i_3 - i_2)\,dt + L_2 \frac{di_3}{dt} + \frac{1}{C_3}\int i_3\,dt$$

The F - V analogous network is shown in the Fig. Q.18.1 (b).

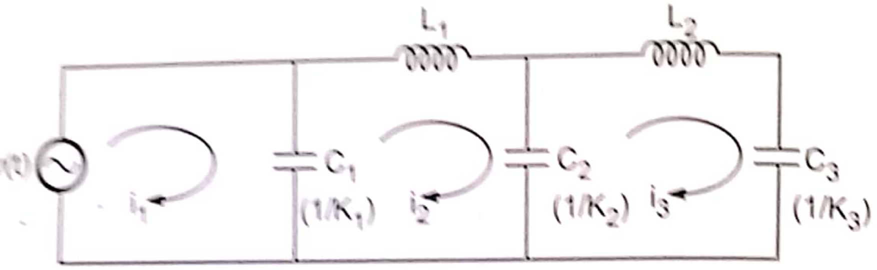

L1
L2
v(t)
C1
(1/K1)
i1
i2
C2
(1/K2)
i3
C3
(1/K3)

Fig. Q.18.1 (b)

Q.19 Obtain a mathematical model for the mech. system shown Write DE, using F-V analogy. Draw equivalent mechanical circuit.

☞ [SPPU : April-17 (In Sem), Marks 6]

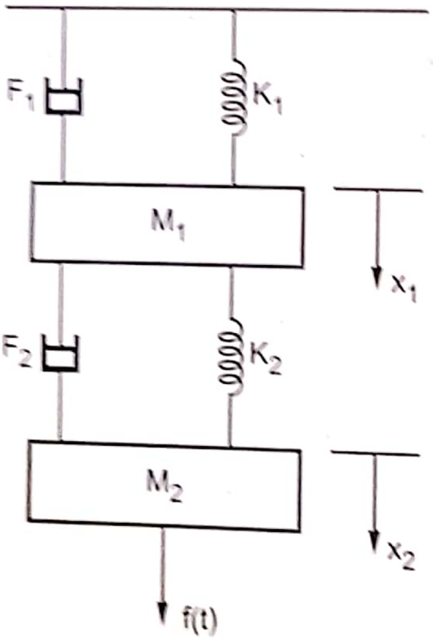

F1
K1
M1
x1
F2
K2
M2
x2
f(t)

Fig. Q.19.1

Ans. : The mass M_2 is under the displacement x_2. The displacement changes from x_2 and x_1 due to K_2 and F_2. While M_1, K_1 and F_1 are under the displacement x_1. Hence the equivalent mechanical system is shown in the Fig. Q.19.1(a).

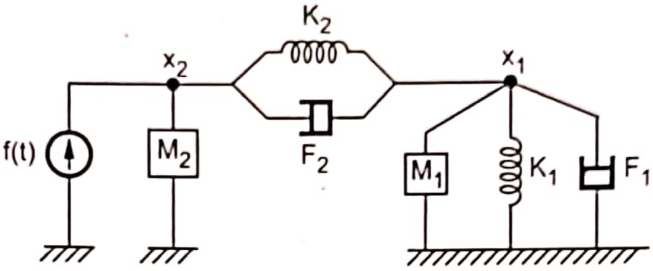


Fig. Q.19.1 (a)

The equilibrium equations are,

$$f(t) = M_2 \frac{d^2x_2}{dt^2} + K_2 (x_2 - x_1) + F_2 \frac{d(x_2 - x_1)}{dt} \quad \dots (1)$$

$$0 = K_2 (x_1 - x_2) + F_2 \frac{d(x_1 - x_2)}{dt} + M_1 \frac{d^2x_1}{dt^2} + K_1 x_1 + F_1 \frac{dx_1}{dt} \quad \dots (2)$$

For F-V electrical analogous network use,

$$M \to L,\ F \to R,\ K \to \frac{1}{C},\ x \to q,\ \dot{x} \to i,\ x \to \int i\,dt,\ \ddot{x} = \frac{di}{dt}$$

$$\therefore \quad v(t) = L_2 \frac{di_2}{dt} + \frac{1}{C_2}\int (i_2 - i_1)\,dt + R_2 (i_2 - i_1) \quad \dots (3)$$

$$\therefore \quad 0 = \frac{1}{C_2}\int (i_1 - i_2)dt + R_2 (i_1 - i_2) + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int i_1\,dt + R_1 i_1 \quad \dots (4)$$

The electrical analogous network is shown in the Fig. Q.19.1(b).

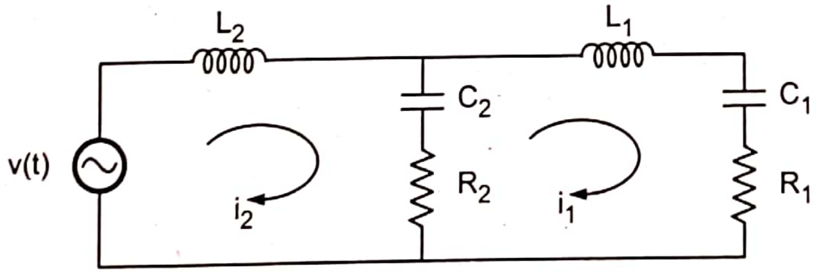


Fig. Q.19.1 (b)

Q.20 Find transfer function of given mechanical system between x_2 and f(t). [SPPU : Dec.-17, Marks 6]

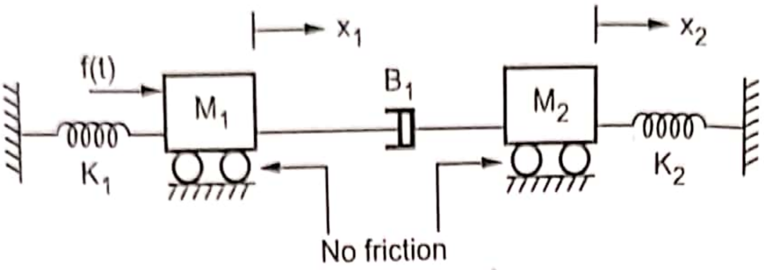


Fig. Q.20.1

Ans. : M_1, K_1 are under x_1, B_1 is between x_1 and x_2 while M_2, K_2 are under x_2.

The equivalent system is shown in the Fig. Q.20.1 (a). The equilibrium equations are,

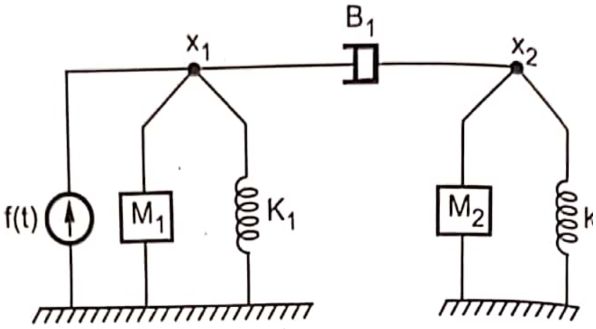


Fig. Q.20.1 (a)

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + K_1 x_1 + B_1 \frac{d(x_1 - x_2)}{dt} \quad \ldots(1$$

$$0 = B_1 \frac{d(x_2 - x_1)}{dt} + M_2 \frac{d^2 x_2}{dt^2} + K_2 x_2 \quad \ldots(2$$

Taking Laplace transform of both equations and neglecting initia conditions,

$$F(s) = M_1 s^2 X_1(s) + K_1 X_1(s) + B_1 s X_1(s) - B_1 s X_2(s)$$

$$0 = B_1 s X_2(s) - B_1 s X_1(s) + M_2 s^2 X_2(s) + K_2 X_2(s)$$

$$\therefore \quad X_1(s) = \frac{X_2(s)[M_2 s^2 + B_1 s + K_2]}{B_1 s}$$

$$\therefore \quad F(s) = [M_1 s^2 + B_1 s + K_1] X_1(s) - B_1 s X_2(s)$$

$$\therefore \quad F(s) = \frac{[M_1 s^2 + B_1 s + K_1][M_2 s^2 + B_1 s + K_2]}{B_1 s} X_2(s) - B_1 s X_2(s)$$

$$\therefore \quad \frac{X_2(s)}{F(s)} = \frac{B_1 s}{(M_1 s^2 + B_1 s + K_1)(M_2 s^2 + B_1 s + K_2) - (B_1 s)^2}$$

Solving,
$$\boxed{\frac{X_2(s)}{F(s)} = \frac{B_1 s}{s^4 M_1 M_2 + s^3 (M_1 + M_2) B_1 + s^2 (K_1 M_2 + K_2 M_1) + s(K_1 + K_2) B_1 + K_1 K_2}}$$

Q.21 **Determine equilibrium equations for following system using force - current analogy.** ☞ [SPPU : April-18, (In Sem), Marks 6]

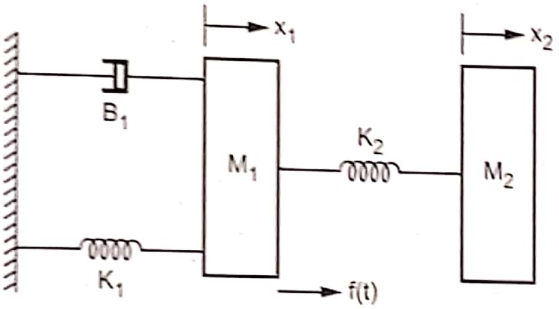


Fig. Q.21.1

Ans. : M_2 is under x_2, K_2 is between x_1 and x_2, M_1, K_1, B_1 are under x_1.

The equivalent system is shown in the Fig. Q.21.1 (a). The equilibrium equations are,

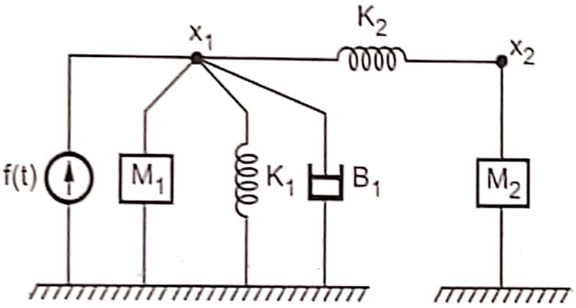


Fig. Q.21.1(a)

$$f(t) = M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + K_2 (x_1 - x_2) \qquad ...(1)$$

$$0 = K_2 (x_2 - x_1) + M_2 \frac{d^2 x_2}{dt^2} \qquad ...(2)$$

For F - I analogy : $M \to C$, $B \to \frac{1}{R}$, $K \to \frac{1}{L}$, $\frac{dx}{dt} \to v(t)$, $\frac{d^2 x}{dt^2} \to \frac{dv(t)}{dt}$,

$x \to \int v(t) dt$

Hence the equations using F - I analogy are,

$$i(t) = C_1 \frac{dv_1(t)}{dt} + \frac{v_1(t)}{R_1} + \frac{1}{L_1} \int v_1(t) dt + \frac{1}{L_2} \int [v_1(t) - v_2(t)] dt$$

$$0 = \frac{1}{L_2} \int [v_2(t) - v_1(t)] dt + C_2 \frac{dv_2(t)}{dt}$$

Q.22 Find transfer function of Fig. Q.22.1.

☞ [SPPU : May-16, Marks 7]

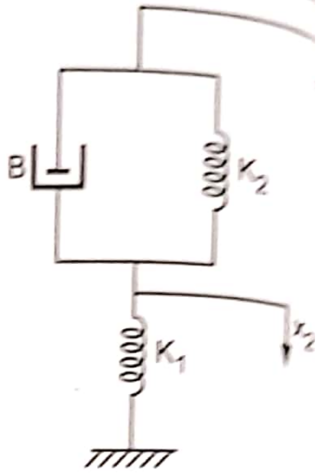


Fig. Q.22.1

Ans. : There are two displacements x_1 and x_2. The input is x_1 and the output is x_2 hence the transfer function is $\frac{X_2(s)}{X_1(s)}$.

The equilibrium equation is,

$$B\frac{d(x_2 - x_1)}{dt} + K_2(x_2 - x_1) + K_1 x_2 = 0$$

Taking Laplace transform,

$$Bs\,X_2(s) - Bs\,X_1(s) + K_2\,X_2(s) - K_2\,X_1(s) + K_1\,X_2(s) = 0$$

$$\therefore \quad X_2(s)\,[Bs + K_2 + K_1] = X_1(s)\,[Bs + K_2]$$

$$\therefore \quad \frac{X_2(s)}{X_1(s)} = \boxed{\frac{Bs + K_2}{Bs + K_1 + K_2}}$$

1.14 : Block Diagram Representation

Important Points to Remember

- The block diagram representation is a pictorial representatio of the given system representing the given complicate practical system in a very simple way.
- In block diagram, each element of practical system i represented by a block. The block is called **functional block** The actual mathematical function is indicated by insertin corresponding transfer function of the element inside th block.
- In a block diagram, the point where addition and subtraction of signals take place is indicated by the **summing point**. The

indication of signs to various signals at the summing point represents addition or subtraction.

- A point in a block diagram from where a signal is taken for the comparison purpose (feedback) is called a **take off point.**
- All the functional blocks, summing points and take off points then are connected exactly as per actual elements connected in the practical system with the help of lines called branches of block diagram.
- An arrow is associated with each and every branch which indicates the direction of flow of signal along the branch.
- The signal can travel along the direction of an arrow only.

Q.23 Derive the transfer function of canonical form of closed loop system. ☞ **[SPPU : May-15, Marks 4]**

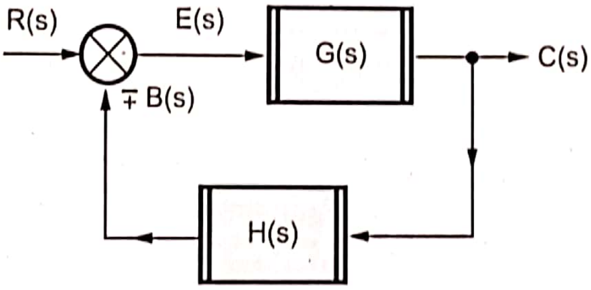


Fig. Q.23.1

Ans. : • A block diagram in which, forward path contains only one block, feedback path contains only one block, one summing point and one take off point represents **simple** or **canonical form** of a closed loop system.

- The simple form can be shown as in the Fig. Q.23.1. From this, we can write the following equations,

$$E(s) = R(s) \mp B(s) \qquad \text{... (1)}$$

$$B(s) = C(s)H(s) \qquad \text{... (2)}$$

$$C(s) = E(s)G(s) \qquad \text{... (3)}$$

Substituting $E(s) = \frac{C(s)}{G(s)}$ from (3) and B(s) from (2) in equation (1),

$$\frac{C(s)}{G(s)} = R(s) \mp C(s)H(s)$$

$$C(s) = R(s)G(s) \mp C(s)G(s)H(s) \quad \text{i.e.} \quad C(s)\,[1 \pm G(s)\,H(s)\,] = R(s)G(s)$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}}$$

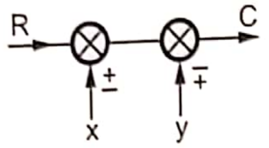
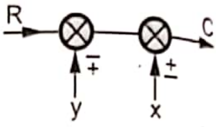
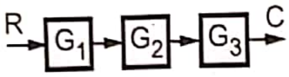
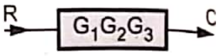
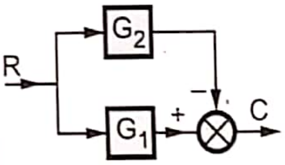
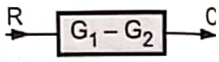
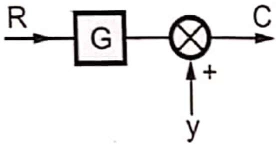
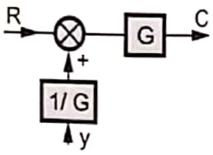
Use + sign for negative feedbac

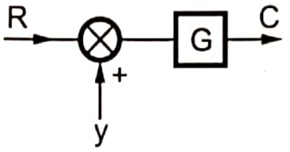
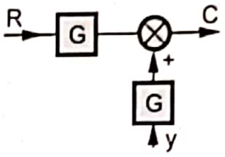
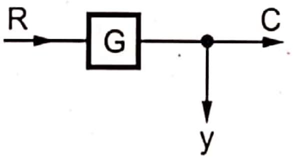
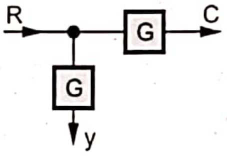
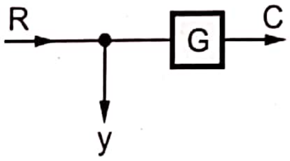
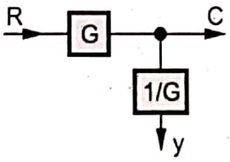
Use – sign for positive feedbac

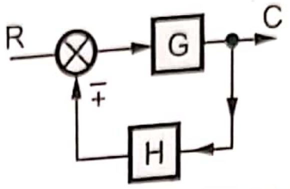
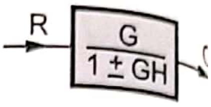
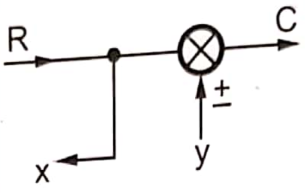
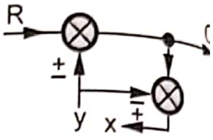
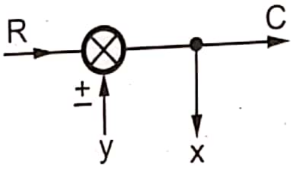
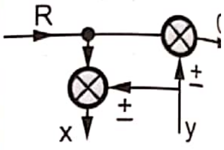
Q.24 Explain the rules of block diagram reduction technique.

☞ [SPPU : Dec.-13, May-14, Marks 6]

Ans. : Table of Block Diagram Reduction Rules.

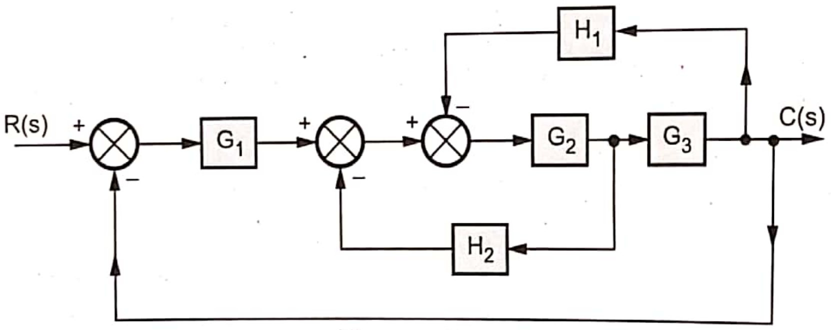
1.	Associative law	The two or more summing points directly connected can be interchanged.		
2.	Blocks in series	Transfer functions of such blocks get multiplied		
3.	Blocks in parallel	Transfer functions of such blocks get added algebraically		
4.	Shifting summing point behind the block	Add a block of T.F. equal to reciprocal of block behind which summing point is to be shifted in series with all signals at that summing point		

5.	Shifting summing point beyond the block	Add a block of T.F. same as the block beyond which summing point is to be shifted, in series with all the signals at that summing point		
6.	Shifting a take-off point behind the block	Add a block of T.F. equal to the block behind which take-off point is to be shifted, in series with all the signals at that takeoff point		
7.	Shifting a take-off point beyond the block	Add a block of T.F. equal to the reciprocal of the block beyond which take-off point is to be shifted, in series with all the signals at that take-off point		

8.	Removing minor feedback loop	Use the standard T.F. of a simple closed loop system		
9.	Shifting take-off point after summing point	Add a new summing point to compensate for the shift of take off point.		
10.	Shifting a take-off point before a summing point	Add a new summing point to compensate for a shift of take-off point		

Q.25 Find transfer function for following Fig. Q.25.1.

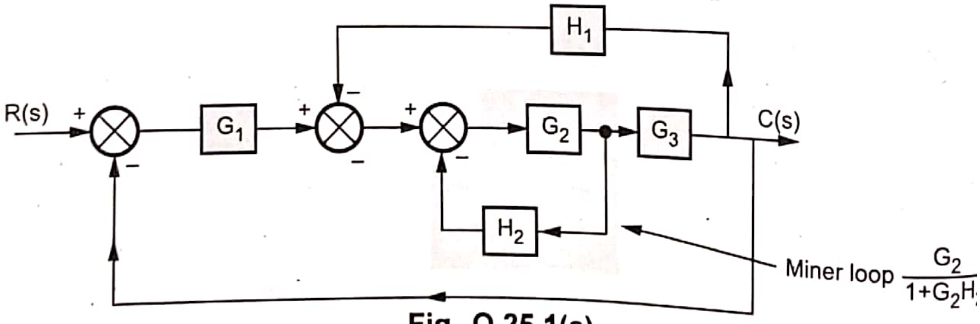
☞[SPPU : April-16, (In Sem), Marks 6]



H1
R(s)
+
−
G1
+
−
+
−
G2
G3
C(s)
H2

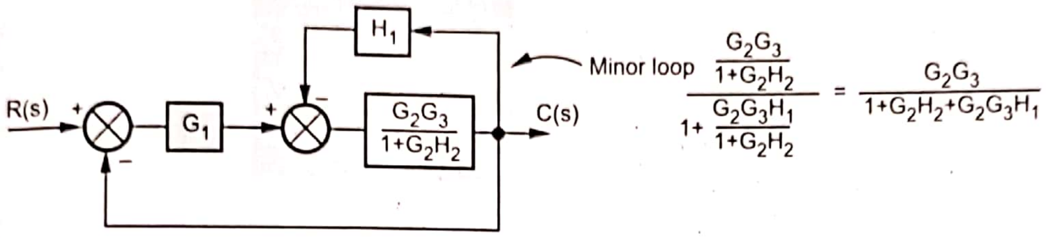
Fig. Q.25.1

Ans. : Exchange the positions of two summing points.



H1
R(s)
+
−
G1
+
−
−
+
−
G2
G3
C(s)
H2
Miner loop
G2
1+G2H

Fig. Q.25.1(a)


R(s)
+
−
G1
+
−
H1
$\frac{G_2G_3}{1+G_2H_2}$
C(s)
Minor loop

$$\frac{\frac{G_2G_3}{1+G_2H_2}}{1+\frac{G_2G_3H_1}{1+G_2H_2}} = \frac{G_2G_3}{1+G_2H_2+G_2G_3H_1}$$

Fig. Q.25.1(b)

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_1}}{1+\frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_1}}$$

$$\therefore \quad \boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2H_2+G_2H_3H_1+G_1G_2G_3}}$$

Q.26 Derive transfer function using block diagram reduction.

☞ [SPPU : May-17, Marks 6]

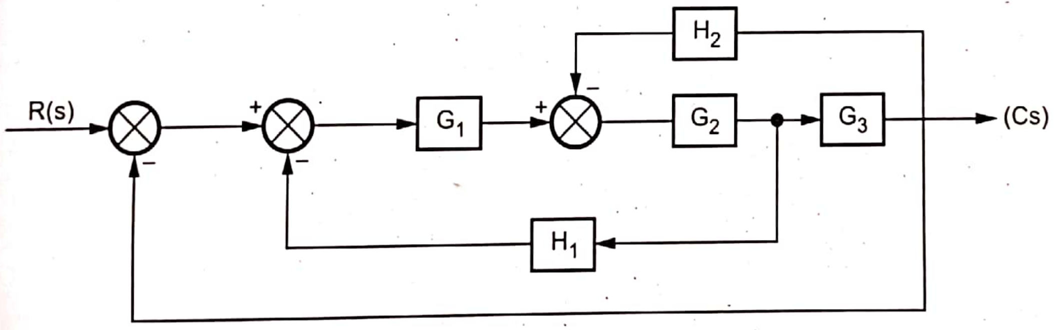
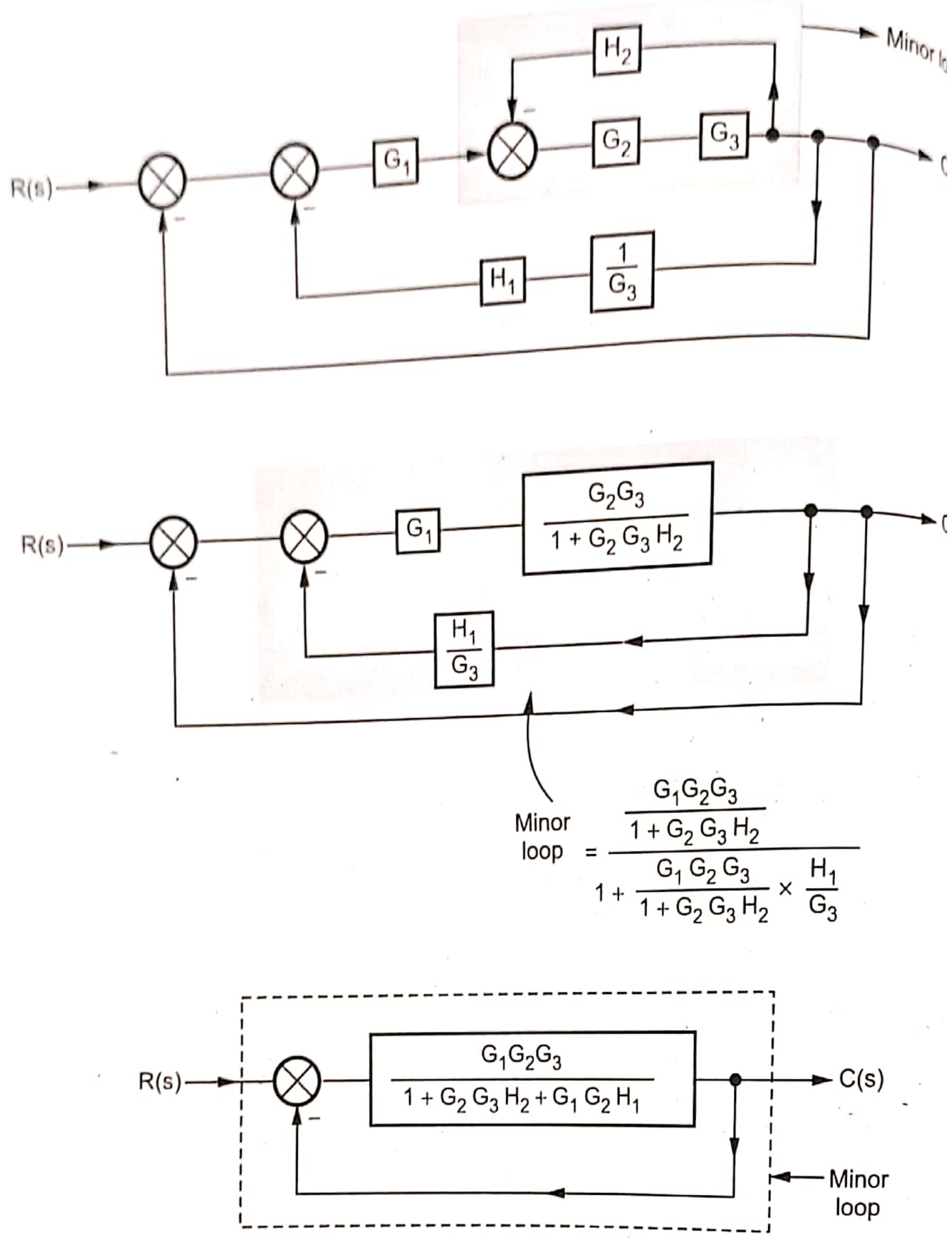

H2
R(s)
+
−
+
−
G1
+
−
G2
G3
(Cs)
H1

Fig. Q.26.1

Ans. : Shift the take off point of H_1 after G_3.


R(s)
H₂
G₁
G₂
G₃
Minor lo
H₁
1/G₃
$\frac{G_2G_3}{1+G_2G_3H_2}$
$\frac{H_1}{G_3}$
$\frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1}$
C(s)
Minor loop

$$\text{Minor loop} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2}}{1+\dfrac{G_1G_2G_3}{1+G_2G_3H_2}\times\dfrac{H_1}{G_3}}$$

Fig. Q.26.1(a)

Using $\frac{G}{1+GH}$ for final minor loop and solving,

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1+G_2G_3H_2+G_1G_2H_1+G_1G_2G_3}}$$

Q.27 Obtain the overall transfer for a system represented by a block diagram. ☞ [SPPU : April-17. (In Sem), Marks 6]

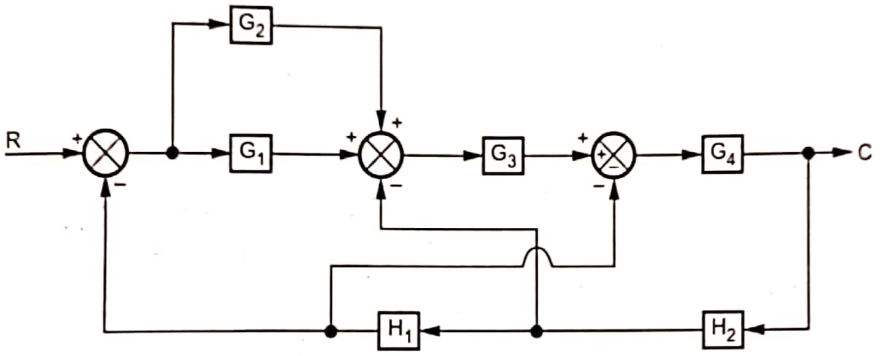

R
+
−
G$_2$
G$_1$
+
+
−
G$_3$
+
−
G$_4$
C
H$_1$
H$_2$

Fig. Q.27.1

Ans. : Combine G_1 and G_2 in parallel. Separate the paths linked through feedback as shown in the Fig. Q.27.1(a).

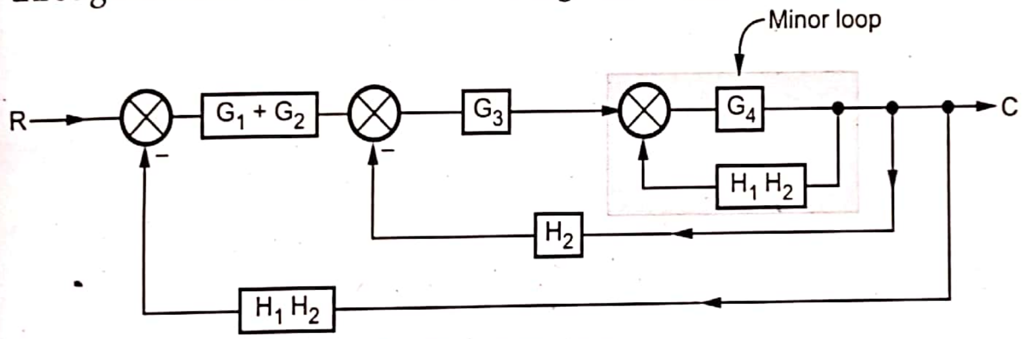
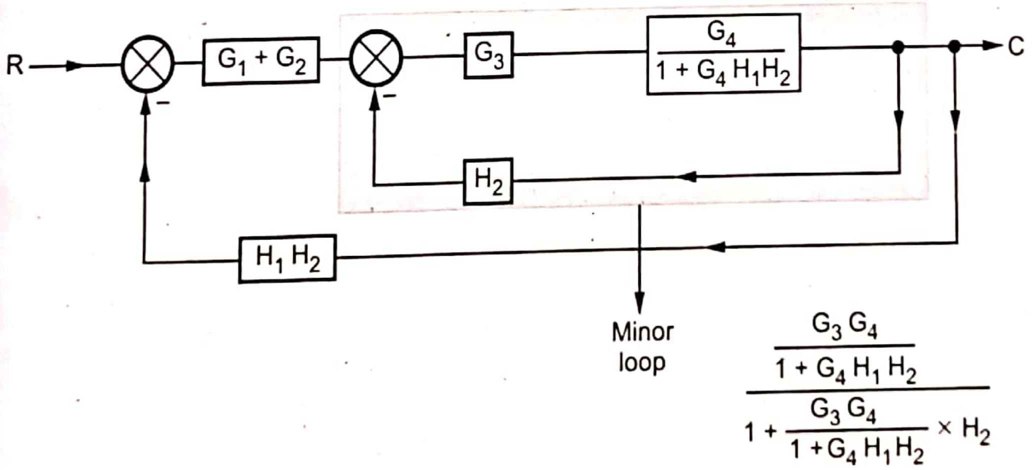

Minor loop
R
−
G$_1$ + G$_2$
−
G$_3$
G$_4$
H$_1$ H$_2$
H$_2$
H$_1$ H$_2$
C

Fig. Q.27.1(a)


R
−
G$_1$ + G$_2$
−
G$_3$
H$_2$
H$_1$ H$_2$
C
Minor loop

$$\frac{\dfrac{G_3 G_4}{1 + G_4 H_1 H_2}}{1 + \dfrac{G_3 G_4}{1 + G_4 H_1 H_2} \times H_2}$$

$$\frac{G_4}{1 + G_4 H_1 H_2}$$

Fig. Q.27.1(b)

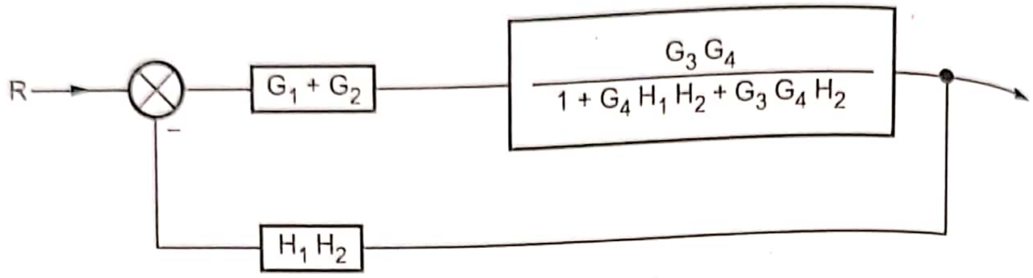

R
–
$G_1 + G_2$
$\frac{G_3 G_4}{1 + G_4 H_1 H_2 + G_3 G_4 H_2}$
$H_1 H_2$

Fig. Q.27.1(c)

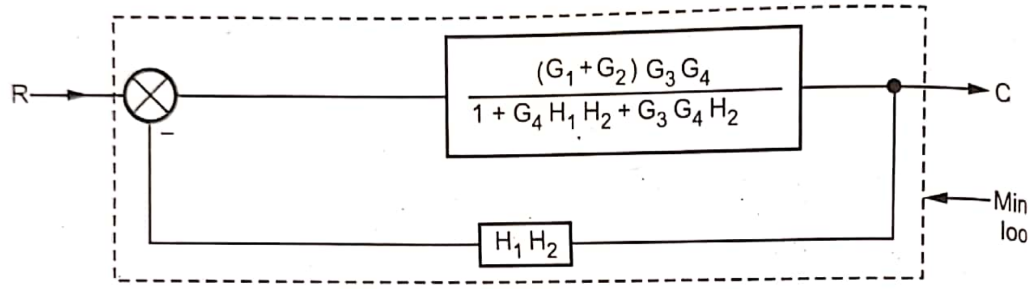

R
–
$\frac{(G_1 + G_2) G_3 G_4}{1 + G_4 H_1 H_2 + G_3 G_4 H_2}$
C
$H_1 H_2$
Min
loo

Fig. Q.27.1(d)

Solving by $\frac{C}{R} = \frac{G}{1+GH}$ we get

$$\boxed{\frac{C}{R} = \frac{(G_1 + G_2) G_3 G_4}{1 + G_4 H_1 H_2 + G_3 G_4 H_2 + (G_1 + G_2) G_3 G_4 H_1 H_2}}$$

Q.28 Using block diagram reduction technique find the transfe function of ☞ **[SPPU : Dec.-17, Marks 6]**

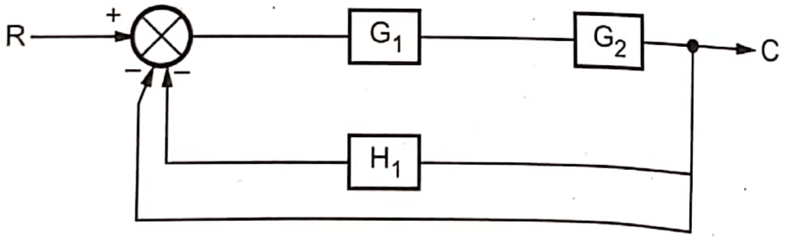


Fig. Q.28.1

Ans. : Separating the paths in feedback,

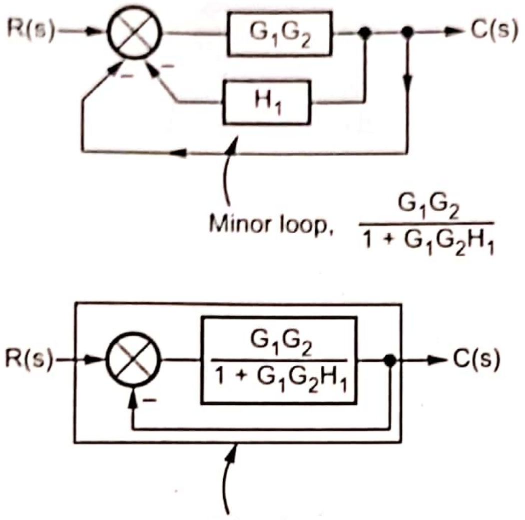

R(s)
G_1G_2
C(s)
H_1
–
–
Minor loop, $\frac{G_1G_2}{1+G_1G_2H_1}$
R(s)
$\frac{G_1G_2}{1+G_1G_2H_1}$
C(s)
–
Minor loop

Fig. Q.28.1(a)

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{G_1G_2}{1+G_1G_2H_1}}{1+\dfrac{G_1G_2}{1+G_1G_2H_1}} = \boxed{\frac{G_1G_2}{1+G_1G_2H_1+G_1G_2}}$$

Q.29 Obtain transfer function of the system.

☞ [SPPU : April-18, (In Sem.) Marks 6]

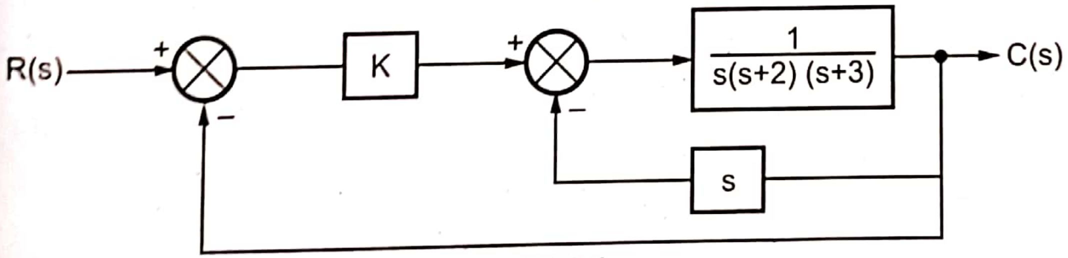

R(s)
+
–
K
+
–
$\frac{1}{s(s+2)(s+3)}$
C(s)
s

Fig. Q.29.1

Ans. : Reduce the minor feedback loop as shown in the Fig. Q.29.1(a).

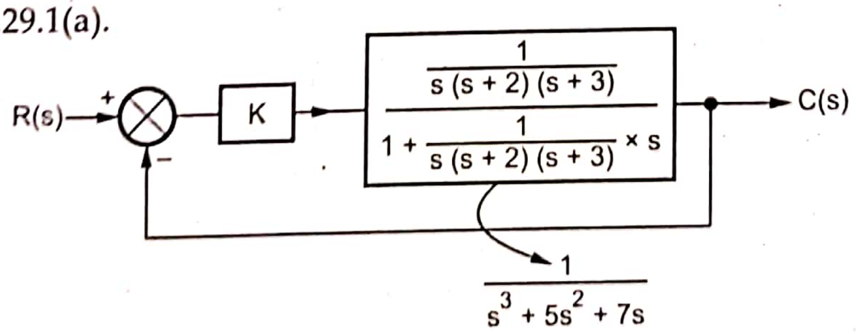

R(s)
+
–
K
$\frac{\frac{1}{s(s+2)(s+3)}}{1+\frac{1}{s(s+2)(s+3)}\times s}$
C(s)
$\frac{1}{s^3+5s^2+7s}$

Fig. Q.29.1 (a)

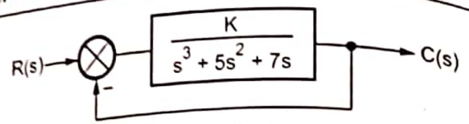


Fig. Q.29.1(b)

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{K}{s^3+5s^2+7s}}{1+\dfrac{K}{s^3+5s^2+7s}} = \boxed{\frac{K}{s^3+5s^2+7s+K}}$$

1.15 : Signal Flow Graph and Mason's Gain Formula

Q.30 Define signal flow graph.

Ans. : • The graphical representation of the variables of a set linear algebraic equations representing the system is called sig flow graph.

- Each variable of a system is represented by a seperate node signal flow graph. The lines joining the nodes are call **branches**. Each branch is associated with the transfer functi and an arrow.
- The transfer function represents mathematical operation on o variable to produce the other variable. The arrow indicates t flow of signal and **signal can travel only along an arrow.**

Q.31 State the properties of signal flow graph.

Ans. : The various properties of signal flow graph are,

1) The signal flow graph is applicable only to **linear time invaria systems.**

2) The signal in the system flows along the branches and along t arrowheads associated with the branches.

3) The signal gets multiplied by the branch gain or bran transmittance when it travels along it.

4) **The value of variable represented by any node is an algebraic sum of all the signals entering at that node.**

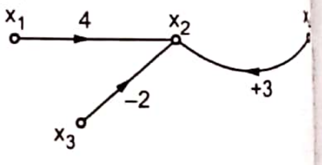


Fig. Q.31.1

- Consider the part of the signal flow graph shown in the Fig. Q.31.1.
- The value of the variable x_2 is $4x_1 - 2x_3 + 3x_4$ which is algebraic sum of all the signals entering the node x_2.

5) The value of the variable represented by any node is available to all the branches leaving that node. **The number of branches leaving a node does not affect the value of variable represented by that node.**

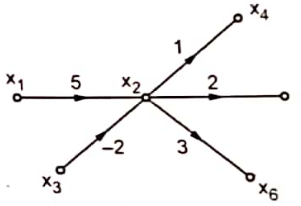


Fig. Q.31.2

- Consider signal flow graph represented in Fig. Q.31.2. The value of x_2 can be obtained from signals entering at x_2.

  i.e. $\quad x_2 = 5x_1 - 2x_3$

- There are three branches leaving x_2, joining to x_4, x_5 and x_6 and same value of x_2 is available to all of them.
- So we can write $x_4 = x_2$, $x_5 = 2x_2$, $x_6 = 3x_2$

6) For a given system signal flow graph is not unique. Many other graphs can be drawn by writing system equations in different manner.

Thus the signal flow graph is not the unique property of the system.

Q.32 Define the following terms with respect to signal flow graph : i) Source node ii) Sink node iii) Chain node iv) Dummy node v) Forward path vi) Feedback loop vii) Loop gain viii) Non-touching loop ix) Self loop. ☞ **[SPPU : Dec.-11, Marks 4]**

Ans. : • Consider a signal flow graph shown in the Fig. Q.32.1

i) **Source node :** The node having only outgoing branches is known as source or input node. e.g. x_0 is source node in the Fig. Q.32.1.

ii) **Sink node :** The node having only incoming branches is known as sink or output node. e.g. x_5 is sink nodein the Fig. Q.32.1.

iii) **Chain node :** A node having incoming and outgoing branches is known as chain node. e.g. x_1 , x_2 , x_3 and x_4 the Fig. Q.32.1.

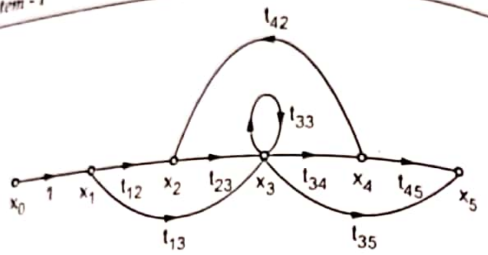


Fig. Q.32.1

iv) **Dummy node :** If seperate source and sink nodes are existing in the signal flow graph then a separate souce and s nodes can be created by adding branches with gain 1 at t start and end. Such nodes are called dummy nodes. Such no can be added only at start and end of signal flow graph and in between the cahin nodes.

v) **Forward path :** A path from the input to output node witho tracing any node twice is defined as forward path. There can many forward paths in a signal flow graph.

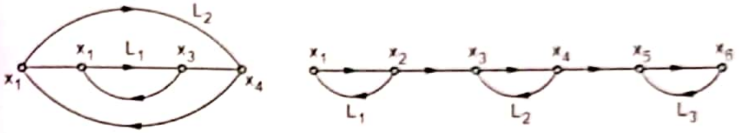
- $x_0 - x_1 - x_2 - x_3 - x_4 - x_5$ is one forward path wh
- $x_0 - x_1 - x_3 - x_4 - x_5$ is another forward path. There are ma other forward paths also.

vi) **Feedback loop :** A path which originates from a particu node and terminating at the same node, travelling through least one other node, **without tracing any node twice** is call feedback loop. For example, $x_2 - x_3 - x_4 - x_2$ in the Fig. Q.32.1

vii) **Loop gain :** The product of all the gains of the branch forming a loop is called loop gain. For example for the lo $x_2 - x_3 - x_4 - x_2$, $t_{23} \times t_{34} \times t_{42}$ is the loop gain.

viii) **Non-touching loops :** If there is no node common in betwe the two or more loops, such loops are said to be non-touchi loops.

- The Fig. Q.32.2 (a) and (b) shows a combination of non-touchi loops of two and three loops.



(a) Two non-touching loops (b) Three non-touching loops

Fig. Q.32.2

ix) **Self loop :** A feedback loop consisting of only one node is called self loop. For example t_{33} at x_3 is a self loop.

- A self loop can not appear while defining a forward path or feedback loop as node containing it gets traced twice which is not allowed.

Q.33 State and explain Mason's gain formula.

☞ [SPPU : April-12, 16,18 (In Sem), Dec.-16, Marks 6]

Ans. : • The Mason's gain formula is used to obtain closed loop transfer function of a system from its signal flow graph. The formula can be stated as,

$$\boxed{\text{Overall closed loop T.F. } \frac{C(s)}{R(s)} = \frac{\Sigma\, T_K \Delta_K}{\Delta}}$$

where K = Number of forward paths,

T_K = Gain of K^{th} forward path

Δ = System determinant to be calculated as :

Δ = 1 − [Σ all individual feedback loop gains [including self loops]] + [Σ Gain × Gain product of all possible combinations of two non-touching loops] − [Σ Gain × Gain × Gain product of combinations of three non touching loops] +.........

Δ_K = Value of above Δ by eliminating all loop gains and associated products which are touching to the K^{th} forward path.

Explanation :

- Let a signal flow graph has 3 forward paths i.e. K = 3.
- The gains of these forward paths be T_1, T_2 and T_3.
- Let there are 3 individual feedback loops including self loop with gains L_1, L_2, and L_3.

- Out of these three, L_1L_2 and L_1L_3 are the combinations of t non-touching loops. No combination of 3 nontouching loops a onwards.

$$\therefore \quad \Delta = 1 - [\Sigma \text{ all individual and self loop gains}] + [\Sigma \text{ gain} \times \text{gain product of all combinations of two non-touching loops}]$$

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_2 + L_1L_3]$$

- For Δ_K i.e. Δ_1, Δ_2 and Δ_3 consider each forward path separate
- For T_1, say all loops L_1, L_2 and L_3 are touching hence all lo gains are to be eliminated from Δ to get Δ_1 i.e. $\Delta_1 = 1$.
- For T_2, say L_2 is non-touching and L_1, L_3 are touching. So l L_3 and associative products i.e. $L_1 L_2$ and L_1L_3 are to eliminated to get Δ_2, L_2 will exist as it is. Hence $\Delta_2 = 1 - L_2$
- For T_3, say L_1 and L_2 both are non-touching and L_3 is touchi So L_3 and associated product L_1L_3 must be eliminated while l L_2 and product L_1 L_2 will exist in Δ_3. He $\Delta_3 = 1 - L_1 - L_2 + L_1 L_2$
- Once all the information is obtained then closed loop trans function can be obtained as,

$$\frac{C(s)}{R(s)} = \frac{\Sigma T_K \Delta_K}{\Delta} \text{ with } K = 3$$

$$\text{i.e. } \frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2 \Delta_2 + T_3 \Delta_3}{\Delta}$$

Q.34 Find transfer function Mason's gain formula following system

☞ [SPPU : April-16 (In Sem), Marks 6

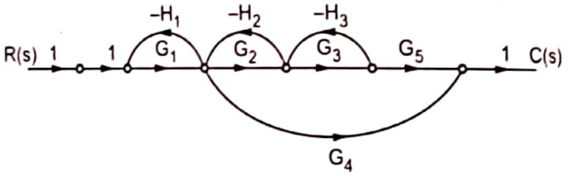


Fig. Q.34.1

Ans. : Two forward paths; $T_1 = G_1G_2G_3G_5$, $T_2 = G_1G_4$

The individual feedback loop gains are,

$L_1 = -G_1 H_1$, $L_2 = -G_2H_2$, $L_3 = -G_3H_3$

Combination of two non-touching loops : L_1L_3

$$\therefore \quad \Delta = 1 - [L_1 + L_2 + L_3] + [L_1L_3]$$

All loops are touching to T_1 hence $\Delta_1 = 1$.

Loop L_3 is non touching to T_2 hence $\Delta_2 = 1 - L_3$.

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{T_1\Delta_1 + T_2\Delta_2}{\Delta} = \boxed{\frac{G_1G_2G_3G_5 + G_1G_4(1 + G_3H_3)}{1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_3H_1H_3}}$$

Q.35 Obtain the transfer function of the signal flow graph given below.

☞ [SPPU : April-15 (In Sem), Marks 6]

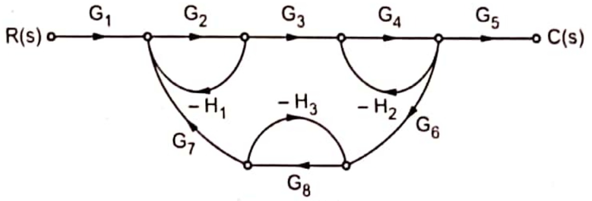
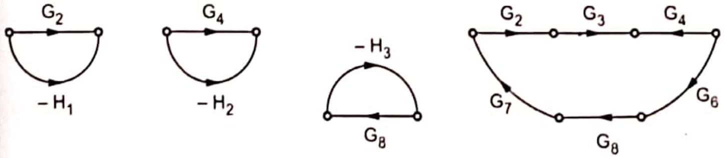


Fig. Q.35.1

Ans. : The forward path gain is,

$$T_1 = G_1 G_2 G_3 G_4 G_5$$

The individual feed back loops are,



$L_1 = -G_2H_1$ $L_2 = -G_4H_2$ $L_3 = -G_8H_3$ $L_4 = G_2G_3G_4G_6G_7G_8$

Fig. Q.35.1(a)

Two nontouching loops : $L_1 L_2$, $L_1 L_3$, $L_2 L_3$

Three nontouching loops : $L_1 L_2 L_3$

$\therefore \Delta = 1 - [L_1 + L_2 + L_3 + L_4] + [L_1L_2 + L_1L_3 + L_2L_3] - [L_1L_2L_3$

Loop L_3 is nontouching to T_1, $\Delta_1 = 1 - L_3$

$$\frac{C(s)}{R(s)} = \frac{T_1\Delta_1}{\Delta}$$

$$\therefore \boxed{\frac{G_1G_2G_3G_4G_5(1-G_8H_3)}{1+G_2H_1+G_4H_2+G_8H_3-G_2G_3G_4G_6G_7G_8+G_2G_4H_1H_2+G_2G_8H_1H_3+G_4G_8H_2H_3+G_2G_4G_8H_1H_2H_3}}$$

Q.36 Obtain the T.F. of given system using Mason's gain formula.

☞ [SPPU : May-03, 07, Marks 10

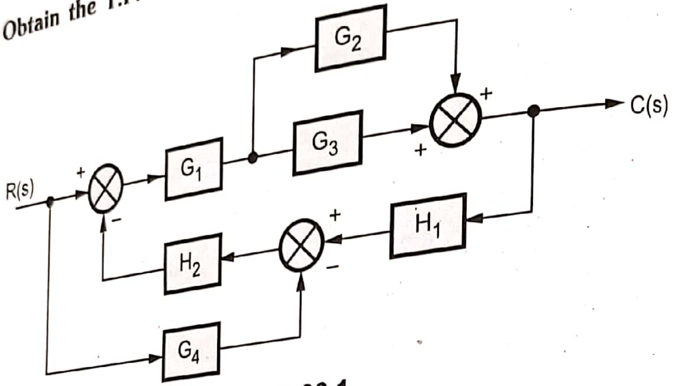
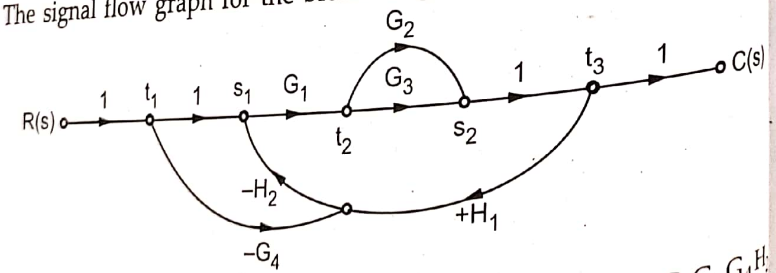


Fig. Q.36.1

Ans. : Name the various summing and take off points. Represe each summing and take off point by separate node.

The signal flow graph for the block diagram is shown in figure.



$T_1 = G_1G_3 \quad T_2 = G_1G_2 \quad T_3 = -G_4 \times -H_2 \times G_1G_3 = G_1G_3G_4H$

$T_4 = -G_4 \times -H_2 \times G_1G_2 = G_1G_2G_4H_2$

$L_1 = -G_1G_3H_1H_2, \qquad L_2 = -G_1G_2H_1H_2$

Both the loops are touching each other so no nontouching loops.

$$\therefore \qquad \Delta = 1-[L_1+L_2]$$

Both the loops are touching to all the forward paths hence,

$$\Delta_1 = \Delta_2 = \Delta_3 = \Delta_4 = 1$$

According to Mason's gain formula,

$$\frac{C(s)}{R(s)} = \frac{\sum_{K=1}^{K} T_K \Delta_K}{\Delta} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3 + T_4\Delta_4}{\Delta}$$

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1 G_3 + G_1 G_2 + G_1 G_3 G_4 H_1 + G_1 G_2 G_4 H_2}{1 + G_1 G_3 H_1 H_2 + G_1 G_2 H_1 H_2}}$$

Q.37 Using Mason's rule, find the transfer function T(s) = C(s)/R(s) for the system represented in the Fig. Q.37.1.

☞ [SPPU : May-18, Marks 7]

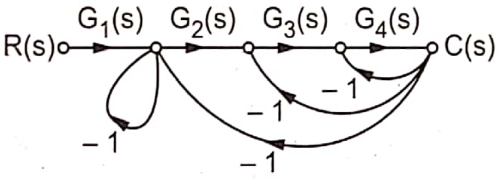


Fig. Q.37.1

Ans. : $T_1 = G_1G_2G_3G_4$

Individual feedback loop gains,

$$L_1 = -1 \text{ (self loop)}, \; L_2 = -G_2 G_3 G_4,$$

$$L_3 = -G_3G_4, \; L_4 = -G_4$$

Combinations of 2 nontouching loops : L_1L_3, L_1L_4,

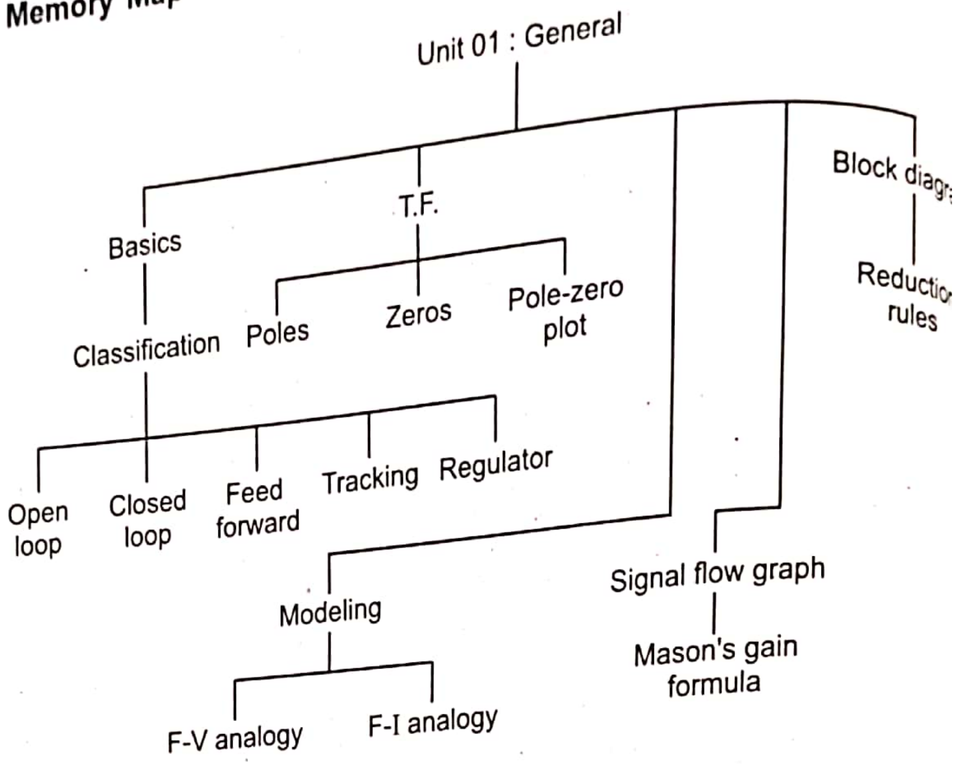
$$\therefore \qquad \Delta = 1-[L_1+L_2+L_3+L_4]+[L_1L_3+L_1L_4]$$

All the loops are touching to T_1 hence $\Delta_1 = 1$.

$$\therefore \qquad T(s) = \frac{T_1\Delta_1}{\Delta} = \frac{G_1G_2G_3G_4}{1+1+G_2G_3G_4+G_3G_4+G_4+G_3G_4+G_4}$$

$$= \boxed{\frac{G_1G_2G_3G_4}{2+G_2G_3G_4+2G_3G_4+2G_4}}$$

Memory Map


Unit 01 : General
Basics
Classification
Open loop
Closed loop
Feed forward
Tracking
Regulator
T.F.
Poles
Zeros
Pole-zero plot
Modeling
F-V analogy
F-I analogy
Signal flow graph
Mason's gain formula
Block diagr
Reductio rules

END...

UNIT - 2

2

Time Domain Analysis

Important Points to Remember

- The response given by the system which is function of the time, to the applied excitation is called **time response** of a control system.
- The output variations during the time it takes to achieve its final value is called **transient response.** The time required to achieve the final value is called transient period.
- The part of the time response which remains after transient response vanishes completely from the system output is called **steady state response.**
- Total time response c(t) = Transient response + Steady state response

i.e. $$c(t) = C_{ss} + c_t(t)$$

- Mathematically for **stable** operating systems, $\lim_{t \to \infty} c_t(t) = 0$
- The difference between the desired output and the actual output of the system is called **steady state error** which is denoted as e_{ss}.

2.1 : Standard Test Signals

Q.1 Discuss the various standard test signals used in control systems along with their equations.

☞ [SPPU : April-15, 16 (In Sem), May-18, (End Sem) Marks 6]

Ans. : The standard test signals are,

i) Step input (Position function) :

- It is the sudden application of the input at a specified time as shown in the Fig. Q.1.1.

- Mathematically it can be described as,

$$\boxed{\begin{aligned} r(t) &= A \quad \text{for } t \geq 0 \\ &= 0 \quad \text{for } t < 0 \end{aligned}}$$

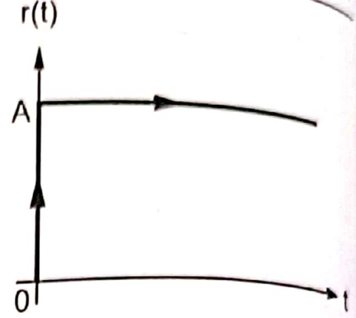


Fig. Q.1.1 Step

- If A = 1, then it is called **unit step function** and denoted by u(t).
- Laplace transform of such input is $\frac{A}{s}$.

ii) Ramp input (Velocity function) :

- It is constant rate of change in input i.e. gradual application of input as shown in the Fig. Q.1.2.
- Magnitude of Ramp input is nothing but its slope. Mathematically it is defined as,

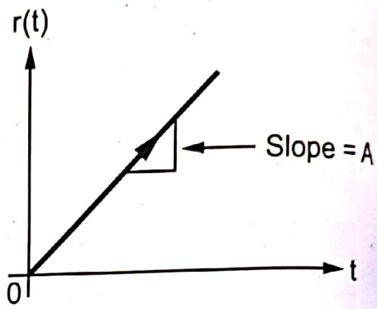


Fig. Q.1.2 Ramp

$$\boxed{\begin{aligned} r(t) &= At \quad \text{for } t \geq 0 \\ &= 0 \quad \text{for } t < 0 \end{aligned}}$$

- If A = 1, it is called **unit ramp input**. It is denoted as r(t). Its Laplace transform is $\frac{A}{s^2}$.

iii) Parabolic input (Acceleration function) :

- This is the input which is one degree faster than a ramp type of input as shown in the Fig. Q.1.3.
- Mathematically this function is described as,

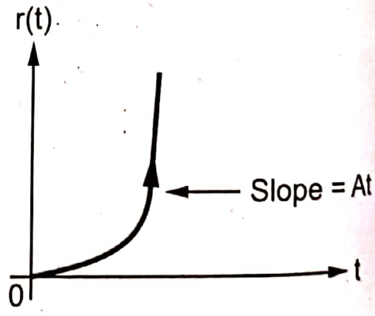


Fig. Q.1.3 Parabolic

$$\therefore \quad \boxed{\begin{aligned} r(t) &= \frac{A}{2}t^2, \quad \text{for } t \geq 0 \\ &= 0, \quad \text{for } t < 0 \end{aligned}}$$

where A is called magnitude of the parabolic input.

Parabolic function is expressed as $\frac{A}{2}t^2$ so that in Laplace transforms of different standard inputs, similarity will get maintained.

- If A = 1, i.e. $r(t) = \frac{t^2}{2}$ it is called **unit parabolic input.** Its Laplace transform is $\frac{A}{s^3}$.

iv) Impulse input :

- It is the input applied instantaneously (for short duration of time) of very high amplitude as shown in the Fig. Q.1.4.
- It is the pulse whose magnitude is infinite while its width tends to zero i.e. $t \to 0$, applied momentarily.
- Area of the impulse is nothing but its magnitude. If its area is unity it is called **unit impulse input**, denoted as $\delta(t)$.
- Mathematically it can be expressed as,

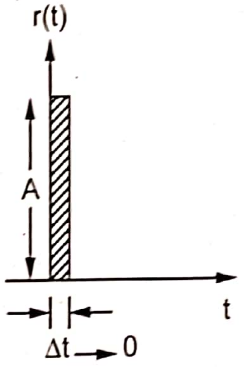


Fig. Q.1.4 Impulse

$$\therefore \quad \boxed{\begin{aligned} r(t) &= A, \quad \text{for } t = 0 \\ &= 0, \quad \text{for } t \neq 0 \end{aligned}}$$

- The Laplace transform of unit impulse input is 1.

2.2 : Steady State Error

Q.2 Derive the expression of steady state error for a simple closed loop system. **☞ [SPPU : April-18, (In sem), Marks 4]**

Ans. : • Consider a simple closed loop system using negative feedback as shown in the Fig. Q.2.1.

$$E(s) = R(s) - B(s)$$

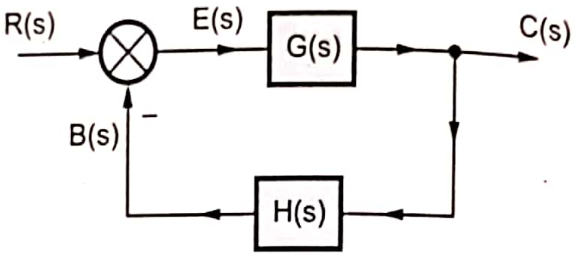


Fig. Q.2.1

But $\quad B(s) = C(s)H(s)$

$\therefore \quad E(s) = R(s) - C(s)H(s)$ and $C(s) = E(s)G(s)$

$\therefore \quad E(s) = R(s) - E(s)\,G(s)H(s)$ i.e. $E(s) + E(s)G(s)H(s) = R(s)$

$$\therefore \quad \boxed{\begin{array}{ll} E(s) = \dfrac{R(s)}{1+G(s)H(s)} & \text{for nonunity feedback} \\ \\ E(s) = \dfrac{R(s)}{1+G(s)} & \text{for unity feedback} \end{array}}$$

- This E(s) is the error in Laplace domain and is expression in 's'. We want to calculate the error value. In time domain, corresponding error will be e(t). Now steady state of the system is that state which remains as t → ∞ .

$$\therefore \quad \boxed{\text{Steady state error, } e_{ss} = \lim_{t \to \infty} e(t)}$$

Using **final value theorem,**

$$\boxed{e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \quad \text{where } E(s) \text{ is } L\{\, e(t) \,\}.}$$

- Substituting E(s) from the expression derived, we can write

$$\boxed{e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}}$$

2.3 : Static Error Coefficients

Q.3 Derive the expressions for static error coefficients ? How these coefficients are related to steady state error ?

☞ **[SPPU : April-17, (In Sem), May-16, (End Sem) Dec.-14, Marks 6]**

Ans. : Consider a system having open loop T.F. G(s)H(s) and excited by various standard test inputs.

a) Reference input is step of magnitude A :

$R(s) = \frac{A}{s}$ and steady state error $e_{ss} = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s\,R(s)}{1+G(s)H(s)}$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot A/s}{1+G(s)H(s)} = \lim_{s \to 0} \frac{A}{1+G(s)H(s)} = \frac{A}{1+\lim_{s \to 0} G(s)H(s)}$$

- For a system selected, $\lim_{s \to 0} G(s)H(s)$ is constant and called **positional error coefficient** of the system denoted as K_p.

$$\boxed{K_p = \lim_{s \to 0} G(s)H(s) = \text{Positional error coefficient}}$$

And corresponding error is,

$$\boxed{e_{ss} = \frac{A}{1+K_p}}$$

b) Reference input is ramp of magnitude `A' :

$R(s) = A/s^2$ and steady state error is $e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}$

$$\therefore\ e_{ss} = \lim_{s \to 0} \frac{s \cdot A/s^2}{1+G(s)H(s)} = \lim_{s \to 0} \frac{A}{s\,[1+G(s)H(s)]} = \lim_{s \to 0} \frac{A}{s+s\,G(s)H(s)}$$

$$\therefore\ e_{ss} = \frac{A}{\lim_{s \to 0} s\,G(s)H(s)}$$

- For a selected system $\lim_{s \to 0} sG(s)H(s)$ is constant and called **velocity error coefficient** as K_v.

$$\boxed{K_v = \lim_{s \to 0} s\, G(s)H(s) = \text{Velocity error coefficient}}$$

And corresponding error is, $\boxed{e_{ss} = \dfrac{A}{K_v}}$

c) Reference input is parabolic of magnitude `A' :

$R(t) = \frac{A}{2} t^2$ i.e. $R(s) = \frac{A}{s^3}$

$$\therefore \qquad e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)} = \lim_{s \to 0} \frac{s \cdot A/s^3}{1+G(s)H(s)}$$

$$= \lim_{s \to 0} \frac{A}{s^2\,[1+G(s)\,H(s)]} = \lim_{s \to 0} \frac{A}{s^2 + s^2\,G(s)H(s)} = \lim_{s \to 0} \frac{A}{s^2\,G(s)H(s)}$$

- So for a selected system $\lim_{s \to 0} s^2\, G(s)H(s)$ is constant and called **acceleration error coefficient** as K_a.

$$\boxed{K_a = \lim_{s \to 0} s^2\, G(s)H(s) = \text{Acceleration error coefficient}}$$

And corresponding error is, $\boxed{e_{ss} = \dfrac{A}{K_a}}$

2.4 : Type and Order of a System

Q.4 What is type and order of a system ?

☞ [SPPU : May-17, (End Sem), Marks 2]

Ans. : • For identifying 'Type' of a system, G(s)H(s) is expressed in a particular form called **time constant form** as given below,

$$G(s)H(s) = \frac{K(1+T_1 s)\,(1+T_2 s)\,\ldots\ldots}{s^j(1+T_a s)\,(1+T_b s)\,\ldots\ldots} \quad \text{where } K = \text{Resultant system gain}$$

- In the above form of G(s)H(s), the value of j indicates the Type of system.
- If j = 0, it is Type 0 system, if j = 1, it is Type 1 system and so on.

- The number of open loop poles located at origin indicates Type of system.
- The highest power of 's' in the characteristic equation of a system is called its **order.**
- Thus type is decided from open loop transfer function while order is decided from closed loop transfer function.

2.5 : Effect of Type of System of Steady State Error

Q.5 Explain the effect of type of a system on steady state error.

☞ [SPPU : May-15, 17, Marks 7]

Ans. : • Consider the input selected as step of magnitude `A'.

i) Type '0' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{(1+T_a s)(1+T_b s)\ldots\ldots}$ **j = 0**

- For step input, $K_p = \lim_{s \to 0} G(s)H(s) = K$ ... Using above G(s)H(s)

$$\therefore \qquad e_{ss} = \frac{A}{1+K_p} = \frac{A}{1+K}$$

- Thus Type `0' systems follow the step type of input with finite error $\frac{A}{1+K}$.

ii) Type '1' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{s\,(1+T_a s)(1+T_b s)\ldots\ldots}$ **j = 1**

- For step input, $K_p = \lim_{s \to 0} G(s)H(s) = \infty$ ... Using above G(s)H(s)

$$\therefore \qquad e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0$$

iii) Type '2' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{s^2\,(1+T_a s)(1+T_b s)\ldots\ldots}$

j = 2

- For step input, $K_p = \lim_{s \to 0} G(s)H(s) = \infty$ ... Using above G(s)H(s)

$$e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = 0$$

∴

- In general, for any Type of system more than zero, K_p will b infinite (∞) and error will be zero for the step input. **Thus Typ 1 and above systems follow a step type of reference input o any magnitude, successfully, with negligibly small error.**
- Consider the input selected as ramp of magnitude 'A'.

i) Type '0' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{(1+T_a s)(1+T_b s)\ldots\ldots}$ $j = 0$

- For ramp input, $K_v = \lim_{s \to 0} s\, G(s)H(s) = 0$ ... Using abov G(s)H(s)
- $\therefore e_{ss} = \dfrac{A}{K_v} = \dfrac{A}{0} = \infty$
- Thus Type '0' systems will not follow ramp input of any magnitude and will give large error in the output.

ii) Type '1' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{s\,(1+T_a s)(1+T_b s)\ldots\ldots}$ $j = 1$

- For ramp input, $K_v = \lim_{s \to 0} s\, G(s)H(s) = K$ ... Using above G(s)H(s)
- $\therefore e_{ss} = \dfrac{A}{K_v} = \dfrac{A}{K}$
- Thus Type '1' systems follow the ramp type of input of magnitude 'A' with finite error A/K.

iii) Type '2' system with $G(s)H(s) = \dfrac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{s^2\,(1+T_a s)(1+T_b s)\ldots\ldots}$ $j = 2$

- For ramp input, $K_v = \lim_{s \to 0} s\, G(s)H(s) = \infty$ ... Using above G(s)H(s)

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{A}{\infty} = 0$$

- Thus Type '2' systems follow the ramp type of input of magnitude 'A' with zero error. **Thus all the systems of Type 2 and more than two follow ramp type of input with negligible small error.**

- Thus it can be seen that for Type 2 system, steady state error is zero for step and ramp inputs while there is finite error for parabolic input.

Q.6 For a unity feedback systems having open loop transfer function as $G(s) = \frac{24\ (s+2)}{s\ (s^2+7s+12)}$ Determine : i) Type of system ii) Error constant K_p, K_v and K_a iii) Steady state error for step input. **☞ [SPPU : April-16, (In Sem), Marks 5]**

Ans. : $$G(s)H(s) = \frac{24(s+2)}{s(s^2+7s+12)} = \frac{24\ (s+2)}{s\ (s+3)(s+4)}$$

$$\therefore G(s)H(s) = \frac{4(1+0.5\,s)}{s\,(1+0.33\,s)(1+0.25\,s)} = \frac{K(1+T_1 s)\ .....}{s^j\ (1+T_a\ s)(1+T_b\ s).....}$$

i) Comparing, j = 1 hence type of system is **one.**

ii) $K_p = \lim_{s \to 0} G(s)(H(s) = \infty$, $K_v = \lim_{s \to 0} G(s)(H(s) = 4$,

$K_a = \lim_{s \to 0} s^2 G(s)(H(s) = 0$.

iii) For step input, $e_{ss} = \frac{A}{1+K_p} = \frac{A}{\infty} = $ **0.**

Q.7 The block diagram of an electronic pacemaker is given in Fig. Q.7.1. Determine the steady state error for unit ramp input when K = 400. Also, determine the value of K for which the steady state error to a unit ramp will be 0.02. **☞ [SPPU : Dec.-15, (End Sem), Marks 7]**

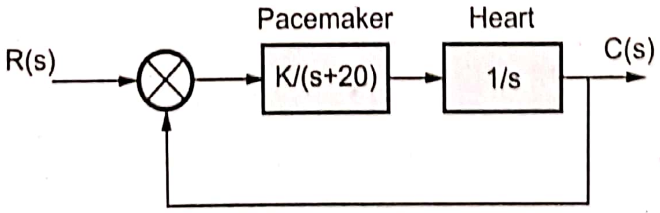


Fig. Q.7.1

Ans. : $$G(s)H(s) = \frac{K}{s(s+20)}$$

For unit ramp input, $$K_v = \lim_{s\to 0} s\,G(s)H(s) = \frac{K}{20}$$

$$K_v = 20$$

i) For $K = 400$,

$\therefore$ $$e_{ss} = \frac{A}{K_v} = \frac{1}{20} = 0.05$$

ii) $e_{ss} = 0.02$ i.e. $0.02 = \frac{1}{K_v}$ i.e. $K_v = 50$

$\therefore$ $50 = \frac{K}{20}$ i.e. $K = 1000$

Q.8 A unity feedback system characterised by the open transfer function.

$$G(s) = \frac{1}{s(0.5s+1)(0.2s+1)}$$

Determine the steady state errors for unit step, unit ramp and acceleration inputs. **[SPPU : Dec.-16, End Sem, Marks**

Ans. : $$G(s)H(s) = \frac{1}{s(1+0.5s)(1+0.2s)}$$

$\therefore$ $$K_p = \lim_{s\to 0} G(s)H(s) = \infty$$

$\therefore$ $$K_v = \lim_{s\to 0} sG(s)H(s) = 1$$

$\therefore$ $$K_a = \lim_{s\to 0} s^2G(s)H(s) = 0$$

For unit step input, unit ramp input and unit acceleration in

A = magnitude = 1

$\therefore$ $$e_{ss} \text{ for unit step} = \frac{A}{1+K_p} = \frac{1}{1+\infty} = 0$$

$\therefore$ $$e_{ss} \text{ for unit ramp} = \frac{A}{K_v} = \frac{1}{1} = 1$$

$\therefore$ $$e_{ss} \text{ for unit acceleration} = \frac{A}{K_a} = \frac{1}{0} = \infty$$

Q.9 A unity feedback system has

$$G(s) = \frac{K}{s(s+1)(1+0.4s)}.$$

i) For ramp input $r(t) = 4t$ and $K = 2$, find steady state error.

ii) If desired steady state error to be 0.2, Find corresponding value of K. ☞ [SPPU : April-18 (In sem), Marks 6]

Ans. :

$$G(s)H(s) = \frac{K}{s(s+1)(1+0.4s)}$$

i)

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{sK}{s(s+1)(1+0.4s)} = K$$

Given K = 2 and r(t) = 4t i.e. A = 4

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{4}{2} = \mathbf{2}$$

ii) Given

$$e_{ss} = 0.2$$

$$\therefore \quad 0.2 = \frac{4}{K_v} \text{ i.e. } 0.2 = \frac{4}{K}$$

$$\therefore \quad K = \mathbf{20}$$

2.6 : Time Response Analysis

Important Points to Remember

Method to Determine Total Output c(t) of a System

1. Determine the closed loop transfer function of the system $\frac{C(s)}{R(s)}$.
2. Find the expression for R(s) from given reference input r(t).
3. Substitute R(s) in the closed loop T.F. to obtain expression for C(s).
4. Take Laplace inverse of C(s) by using partial fraction method to obtain total c(t) of the system to the applied input r(t).

- The roots of the equation 1 + G(s)H(s) = 0 are called **closed loop poles** of the system. The closed loop poles decide the nature of total output c(t) of the system. Hence this equation is called **characteristic equation** of the system.
- The highest power of 's' present in the characteristic equation of the system is called **order** of the system.

2.7 : Analysis of First Order System

Q.10 Obtain the unit step response of a first order system and explain its features. ☞ **[SPPU : May-17, (End Sem), Marks 7]**

OR Write the expression for the closed loop T.F. for first order system and the effect of time constant on system performance. ☞ **[SPPU : April-17, (In Sem), Marks 4]**

Ans. : If the equation 1 + G(s)H(s) = 0 is a first order equation then the system is called first order system. It has only one closed loop pole. Consider a general first order system with closed loop transfer function,

$$\frac{C(s)}{R(s)} = \frac{K}{(s+T)}$$

- For unit step input, r(t) = u(t) and $R(s) = \frac{1}{s}$.

$$\therefore C(s) = \frac{K}{s(s+T)} = \frac{(K/T)}{s} - \frac{(K/T)}{s+T} \quad \text{... Partial fraction}$$

$$\therefore c(t) = L^{-1}[C(s)] = \frac{K}{T} - \frac{K}{T}e^{-Tt}$$

- The response is shown in the Fig. Q.10.1 which is purely exponential.
- The rate of increase depends on the term e^{-Tt} which inturn depends on the value of T.

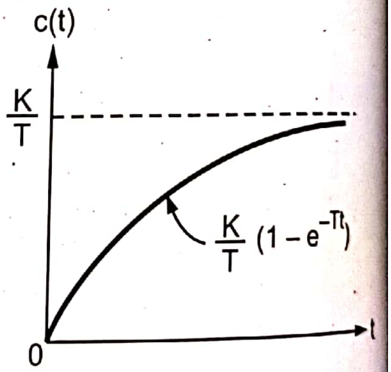


Fig. Q.10.1

Thus closed loop pole s = – T, affects the output response of first order system.

- As closed loop pole moves away from the imaginary axis in left half of s-plane, the transient term e^{-Tt} dies out more quickly, making system more stable.
- The **time constant** of a system is defined as the time required by the system output to reach 63.2 % of its final steady state value during first attempt. It is denoted as τ.

- Practically system settles down in 4 to 5 times the time constant of a system.

2.8 : Analysis of Second Order System

Important Points to Remember

- Every system has a tendency to oppose the oscillatory behaviour of the system which is called **damping**. This damping is measured by a factor or a ratio called **damping ratio** of the system denoted as ξ..
- The frequency of oscillations with which system will oscillate under zero damping ($\xi = 0$) is called natural frequency of oscillations denoted as ω_n rad / sec.
- For a second order system the denominator of closed loop T.F. is quadratic and the coefficients of this equation are directly related to ξ and ω_n.
- The C.L.T.F. (**Closed Loop Transfer Function**) for a standard second order system takes the form as,

$$\boxed{\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\,\omega_n\, s + \omega_n^2} \quad \text{... Standard } 2^{\text{nd}} \text{ order system}}$$

- The characteristic equation is, $s^2 + 2\xi\,\omega_n\, s + \omega_n^2 = 0$.
- If the numerator in closed loop T.F. is other than ω_n^2, but the denominator is quadratic then the system is still second order and its ξ and ω_n can be obtained by comparing its characteristics equation with $s^2 + 2\xi\,\omega_n\, s + \omega_n^2$.
- Always compare denominator of a T.F. with $s^2 + 2\xi\,\omega_n s + \omega_n^2 = 0$ to decide the values of ξ and ω_n of the system.
- The closed loop poles of second order system i.e. roots of characteristic equation are $s_{1,2} = -\xi\,\omega_n \pm j\,\omega_n\sqrt{1-\xi^2}$ where $\omega_n\sqrt{1-\xi^2}$ is called **damped frequency of oscillations** denoted as ω_d rad/sec.

Q.11 Draw time response of second order system for i) $\xi = 1$ ii) $\xi < 1$ iii) $\xi > 1$ iv) $\xi = 0$ Comment on response of system from that.

☞ **[SPPU : Dec.-17, (End Sem), Marks 6]**

OR Explain effect of ξ (damping factor) on locations of closed loop poles.

☞ **[SPPU : April-18 (In Sem), Marks 4]**

Ans. : For a second order system, the roots of characteristic equation are,

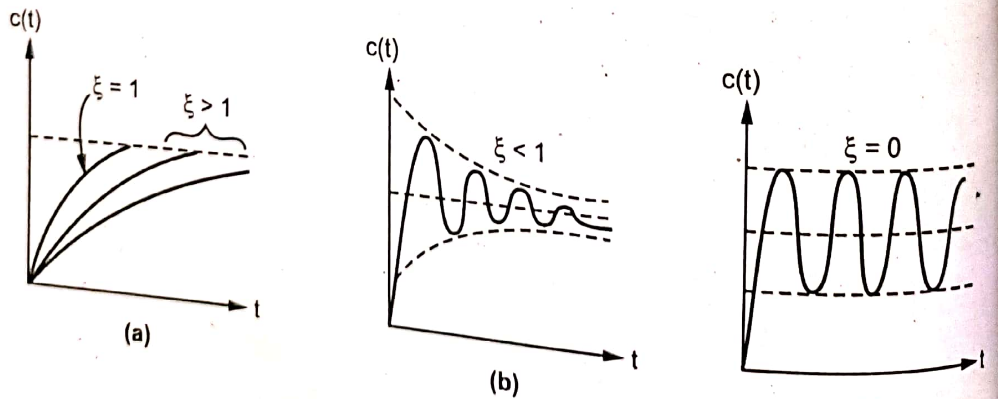
$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

i) For $\xi = 1$, the roots are equal, real and negative. Hence the nature of response is purely exponential with minimum settling time. This is shown in the Fig. Q.11.1(a).

ii) For $\xi < 1$, the roots are complex conjugates with negative real part. Hence the nature of response is oscillations of decreasing amplitude i.e. damped oscillations as shown in the Fig. Q.11.1(b).

iii) For $\xi > 1$, the roots are real, negative and unequal. Hence the nature of response is purely exponential but very slow and sluggish compared to that of $\xi = 1$. This is shown in the Fig. Q.11.1 (a).

iv) For $\xi = 0$, the roots are purely imaginary hence nature of response is oscillations with constant frequency and amplitude. This is shown in the Fig. Q.11.1 (c).



(a) (b) (c)

Fig. Q.11.1

- Thus system with $\xi = 0$ is called **undamped**, with $\xi = 1$ is called **critically damped**, with $\xi > 1$ is called **over damped** while with $\xi < 1$ is called **under damped** system.

2.9 : Unit Step Response of Underdamped Second Order System

Important Points to Remember

- Practically slightly underdamped systems are preferred. Hence range of damping ratio is $0 < \xi < 1$.
- The unit step response of underdamped standard second order system is given by,

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

$$\omega_d = \omega_n \sqrt{1-\xi^2} \quad \text{and} \quad \theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi^2}}{\xi}\right\} \text{ radians}$$

- The response is damped oscillations i.e. oscillations of decreasing amplitude.

2.10 : Time Domain Specifications of Second Order System

Q.12 Define and explain time domain specifications of underdamped second order system with neat sketch.

☞ **[SPPU : May-15, April-16, (In Sem), Dec.-16, (End Sem), Marks 7]**

Ans. : • The response of underdamped second order system for a unit stpe input is shown in the Fig. Q.12.1.

1) Delay time T_d : It is the time required for the response to reach 50 % of the final value in the first attempt. It is given by,

$$\boxed{T_d = \frac{1+0.7\xi}{\omega_n}}$$

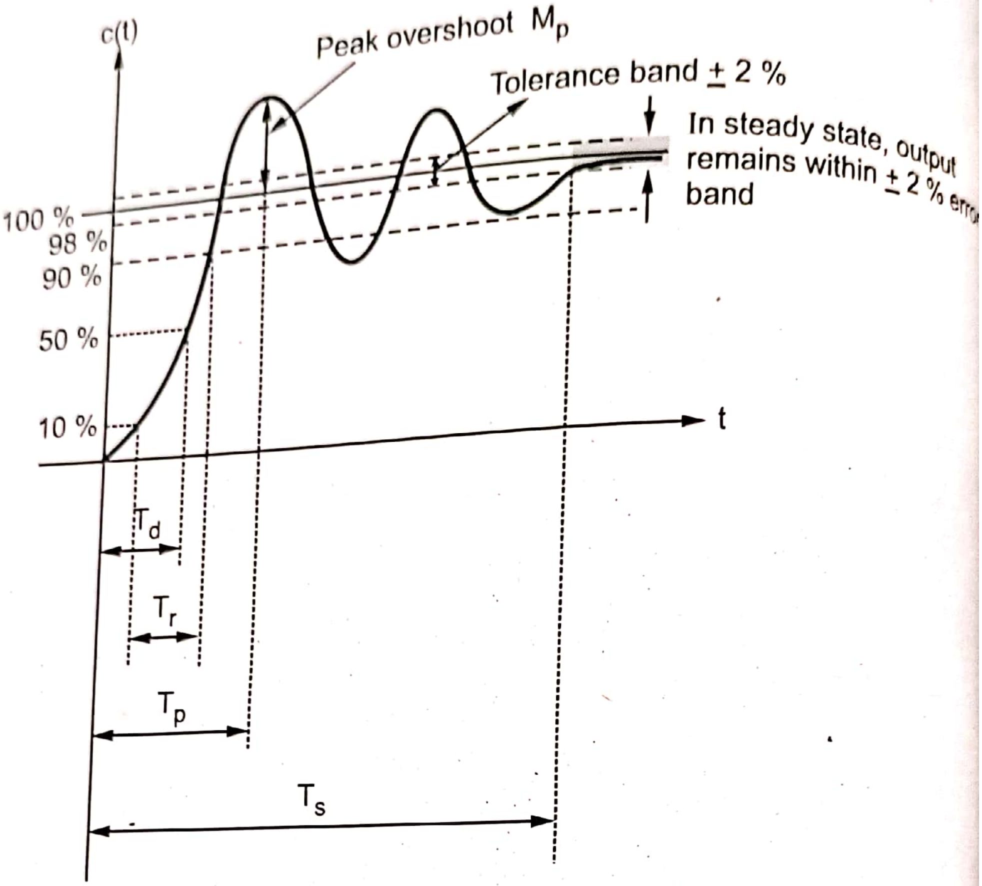

c(t)
Peak overshoot M_p
Tolerance band ± 2 %
In steady state, output remains within ± 2 % error band
100 %
98 %
90 %
50 %
10 %
t
T_d
T_r
T_p
T_s

Fig. Q.12.1 Transient response specifications

2) Rise time T_r : It is the time required for the response to rise from 10 % to 90 % of the final value for overdamped systems and 0 to 100 % of the final value for underdamped systems. The rise time is reciprocal of the slope of the response at the instant, the response is equal to 50 % of the final value. It is given by,

$$\boxed{T_r = \frac{\pi - \theta}{\omega_d} \text{ sec} \quad \text{where } \theta \text{ must be in radians.}}$$

3) Peak time T_p : It is the time required for the response to reach its peak value. It is also defined as the time at which response undergoes the first overshoot which is always peak overshoot.

$$\boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \text{ sec}}$$

4) ***Peak overshoot M_p :*** It is the largest error between reference input and output during the transient period.

It also can be defined as the amount by which output overshoots its reference steady state value during the first overshoot.

$$M_p = \left\{ c(t)\big|_{t = T_p} \right\} - 1 \qquad \text{1 is for unit step input}$$

$$\boxed{\% M_p = e^{-\pi \xi / \sqrt{1-\xi^2}} \times 100}$$

5) ***Settling time T_s :*** This is defined as the time required for the response to decrease and stay within specified percentage of its final value (within tolerance band generally specified as ± 2 % of steady state).

$$\text{Time constant of system} = \frac{1}{\xi \omega_n} = T$$

- Practically the setting time is assumed to be 4 times, the time constant of the system i.e. $T_s = 4 \times \text{Time constant}$

$$\boxed{T_s = \frac{4}{\xi \omega_n} \quad \text{..... For a tolerance band of ± 2 % of steady state}}$$

One time constant 'T' is the time required by the system output to reach 63.2 % of its final value during the first attempt.

2.11 : Derivation of Time Domain Specifications for Second Order System

Q.13 Derive the expressions for the peak time, peak overshoot and rise time for an underdamped second order system.

☞ [SPPU : May-18, (End Sem), Dec.-18, Marks 6]

Ans. :

1. Peak time : The time response of underdamped second order system is,

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) \quad \text{Where} \quad \theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi}$$

- At $t = T_p$, c(t) will achieve its maxima. Hence according to Maxima theorem, $\left.\frac{dc(t)}{dt}\right|_{t=T_p} = 0$.
- So differentiating c(t) w.r.t. 't' we can write,

$$-\frac{e^{-\xi \omega_n t}(-\xi \omega_n)\sin(\omega_d t + \theta)}{\sqrt{1-\xi^2}} - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \omega_d \cos(\omega_d t + \theta) = 0$$

- Substituting $\omega_d = \omega_n \sqrt{1-\xi^2}$ and cancelling $\omega_n e^{-\xi \omega_n t}$,

$$\therefore \quad \xi \sin(\omega_d t + \theta) - \sqrt{1-\xi^2} \cos(\omega_d t + \theta) = 0$$

$$\text{i.e.} \quad \tan(\omega_d t + \theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\text{Now} \quad \theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi} \quad \text{i.e.} \quad \frac{\sqrt{1-\xi^2}}{\xi} = \tan\theta .$$

$$\therefore \quad \tan(\omega_d t + \theta) = \tan\theta \quad \text{i.e.} \quad \omega_d t = n\pi \qquad \text{where } n = 1, 2, 3$$

- But T_p is time required for first peak overshoot. $\therefore$ $n = 1$

$$\omega_d T_p = \pi \quad \text{i.e.} \quad \boxed{T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}} \text{ sec}}$$

2. Peak overshoot (M_p) :

- From Fig. Q.13.1,

$$M_p = C(T_p) - 1$$

$$M_p = \left\{1 - \frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)\right\} - 1$$

$$\therefore M_p = -\frac{e^{-\xi \omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\omega_d T_p + \theta)$$

- But $T_p = \frac{\pi}{\omega_d}$, substituting above,

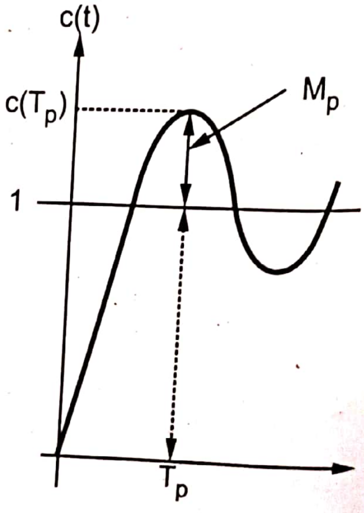


Fig. Q.13.1

$$\therefore \quad M_p = \frac{-e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin(\pi+\theta)$$

$$\therefore \quad M_p = \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \sin\theta \quad \ldots \sin(\pi+\theta) = -\sin(\theta)$$

But $\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi}$ hence $\sin\theta = \sqrt{1-\xi^2}$,

$$\therefore \quad M_p = \frac{e^{-\xi\omega_n T_p}}{\sqrt{1-\xi^2}} \cdot \frac{\sqrt{1-\xi^2}}{1} = e^{-\xi\omega_n T_p}$$

and use $T_p = \frac{\pi}{\omega_d}$

$$\therefore \quad M_p = e^{-\xi\omega_n \frac{\pi}{\omega_n\sqrt{1-\xi^2}}} = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}}$$

i.e. $$\boxed{\% M_p = e^{\frac{-\pi\xi}{\sqrt{1-\xi^2}}} \times 100}$$

3. Rise time (T_r) : Time required by output to achieve 100 % of its final value, starting from zero during the first attempt is the rise time for underdamped system.

$\{c(t)|_{t=T_r}\} = 1$ for unit step input

$$\therefore 1 = 1 - \frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta)$$

$$\therefore \quad -\frac{e^{-\xi\omega_n T_r}}{\sqrt{1-\xi^2}} \sin(\omega_d T_r + \theta) = 0$$

- Equation will get satisfied only if, $\sin(\omega_d T_r + \theta) = 0$

Trigonometrically this is true only if, $\omega_d T_r + \theta = n\pi$ where $n = 1, 2 \ldots$.

- As we are interested in first attempt use $n = 1$

$$\therefore \quad \omega_d T_r + \theta = \pi \quad \text{i.e.} \quad \boxed{T_r = \frac{\pi-\theta}{\omega_d} \text{ sec}}$$

Q.14 **For unity feedback system $G(s) = \dfrac{100}{s^2+15s+100}$ find i) Peak overshoot ii) Settling time iii) Peak time**

☞ **[SPPU : Dec.-17 (End Sem), Marks 4]**

Ans. : The closed loop T.F. is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

$$= \frac{\dfrac{100}{s^2+15s+100}}{1+\dfrac{100}{s^2+15s+100}}$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{100}{s^2+15s+200}$$

Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$, $\omega_n^2 = 200$, $\omega_n = \sqrt{200}$, $2\xi\omega_n = 15$, $\xi = 0.5303$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 11.99 \text{ rad/sec}$$

i) $$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = \mathbf{14.015\ \%}$$

ii) $$T_s = \frac{4}{\xi\omega_n} = \mathbf{0.533\ sec}$$

iii) $$T_p = \frac{\pi}{\omega_d} = \mathbf{0.262\ sec}$$

Q.15 **For system having $\dfrac{C(s)}{R(s)} = \dfrac{20}{s^2+7s+25}$, determine all time domain specifications.**

☞ **[SPPU : April-18 (In sem), Marks 6]**

Ans. : Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$

$\omega_n^2 = 25$, $\omega_n = 5$ rad/sec., $2\xi\omega_n = 7$ i.e. $\xi = 0.7$

$$\therefore \quad \omega_d = \omega_n\sqrt{1-\xi^2} = \mathbf{3.57\ rad/sec}$$

$$\theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = \mathbf{0.795\ rad}$$

$$T_d = \frac{1+0.7\xi}{\omega_n} = \mathbf{0.298\ sec,}$$

$$T_r = \frac{\pi-\theta}{\omega_d} = 0.657 \text{ sec}$$

$$T_p = \frac{\pi}{\omega_d} = 0.88 \text{ sec}, \quad T_s = \frac{4}{\xi\omega_n} = 1.1428 \text{ sec}$$

$$\%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 4.598\ \%$$

Q.16 Find damping ratio, settling time and peak overshoot for system with close loop transfer function given by $T(S) = \frac{10}{s^2+7s+20}$.

☞ [SPPU : May-16 (End Sem), Marks 7]

Ans. : Comparing denominator with $s^2 + 2\xi\,\omega_n s + \omega_n^2$,

$\omega_n^2 = 20,\ \omega_n = 4.4721, \quad 2\xi\omega_n = 7, \ \xi =$ **0.7826**

$$T_s = \frac{4}{\xi\omega_n}$$

$$= \frac{4}{0.7826 \times 4.4721} = \mathbf{1.143\ sec.}$$

$$\%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = \mathbf{1.926\%}$$

Q.17 A unity feedback system is characterised by an open loop.
TF = K/s (s + 10)
Determine the gain K so that the system will have a damping ratio of 0.5. For this value of K determine settling time, peak overshoot and time to peak overshoot for a unit step input.

☞ [SPPU : April-17 (In sem), Dec.-15 (End Sem), Marks 6]

Ans. : The closed loop T.F. is,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K}{s(s+10)}}{1+\frac{K}{s(s+10)}}$$

$$= \frac{K}{s^2+10s+K} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$\therefore\ \omega_n^2 = K,\ \omega_n = \sqrt{K}, \quad 2\xi\omega_n = 10,\ \xi = \frac{5}{\sqrt{K}}$

But $\xi = 0.5$ (given) i.e. $0.5 = \frac{5}{\sqrt{K}}$

$$\therefore \quad K = 100 \quad \text{i.e.} \quad \omega_n = 10 \text{ rad/sec}$$

$$\omega_d = \omega_n \sqrt{1-\xi^2} = 8.66 \text{ rad/sec}$$

$$\therefore \quad T_s = \frac{4}{\xi\omega_n} = 0.8 \text{ sec,}$$

$$T_p = \frac{\pi}{\omega_d} = 0.363 \text{ sec}$$

$$\% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 16.3 \%$$

Q.18 A unity feedback system is characterized by an open loop transfer function

$G(s) = \frac{K}{s(s+10)}$. Determine the gain K so that the system will have damping ratio 0.7 from this value of K determine settling time, peak overshoot and time to peak overshoot.

☞ [SPPU : April-16 (In sem), Marks 6]

Ans. : Refer Q.17 for the procedure and verify the answers as,

$K = 51.02$, $T_s = 0.8$ sec, $T_P = 0.616$ sec, $\% M_P = 4.6\%$.

Q.19 The block diagram for a unity feedback control system is shown in Fig. Q.19.1.

Determine the characteristic equation of the system, ω_n, ξ, ω_d, t_p, M_P, the time at which the first overshoot occurs, the time period of oscillations and the number of cycles completed before reaching the steady state.

☞ [SPPU : April-17 (In sem), Marks 6]

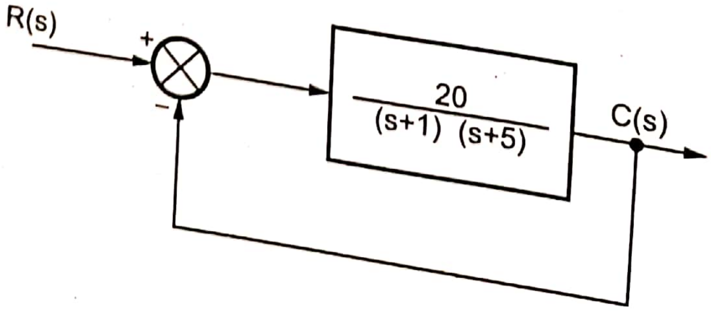


Fig. Q.19.1

Ans. : From given system,

$$\frac{C(s)}{R(s)} = \frac{\frac{20}{(s+1)(s+5)}}{1+\frac{20}{(s+1)(s+5)}} = \frac{20}{s^2+6s+25}$$

Comparing denominator with $s^2 + 2\xi\omega_n s + \omega_n^2$, $\omega_n^2 = 25$, $\omega_n = 5$ **rad/sec**, $2\xi\omega_n = 6$, $\xi = 0.6$

Characteristic equation : $s^2 + 6s + 25 = 0$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4 \text{ rad/sec}$$

$$T_p = \frac{\pi}{\omega_d} = 0.785 \text{ sec}$$

$$\%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = 9.478\ \%$$

$$t = \frac{n\pi}{\omega_d}$$

and at n = 2 first undershoot occurs as shown in the Fig. Q.19.2.

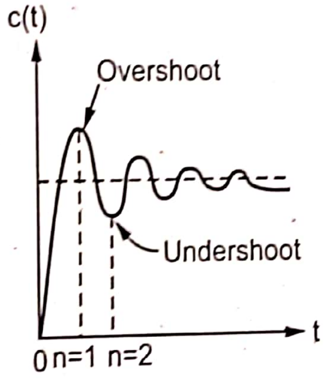


Fig. Q.19.2

$\therefore t = \frac{2\times\pi}{4} = 1.57$ sec ...for 1st undershoot

$\omega_d = \frac{2\pi}{T}$ i.e. T = 1.57 sec/cycle ... Time period

$T_s = \frac{4}{\xi\omega_n} = 1.333$ sec

Thus 1.57 seconds per cycle and system settles in 1.333 seconds hence number of cycles completed before reaching steady state are,

$$\text{Number of cycles} = \frac{1.333}{1.57} = \mathbf{0.849}$$

Q.20 For a control system shown in Fig. Q.20.1, find the values of K_1 and K_2 so that M_p = 25 % and T_p = 4 sec. Assume unit step input. **[SPPU : Dec.-02, May-07, Marks 8]**

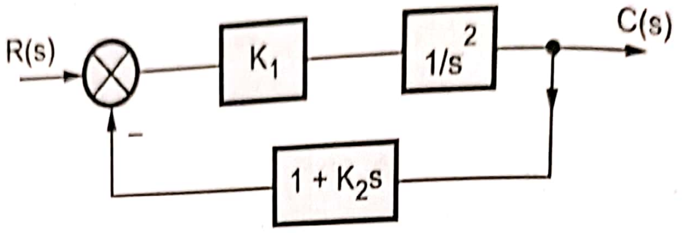


Fig. Q.20.1

Ans. :

$$G(s) = \frac{K_1}{s^2},\quad H(s) = 1 + K_2 s$$

$$\text{T.F.} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{K_1}{s^2}}{1+\frac{K_1}{s^2}(1+K_2\, s)}$$

$$= \frac{K_1}{s^2 + K_1 K_2\, s + K_1}$$

∴ Comparing with standard form,

$$\omega_n = \sqrt{K_1},$$

$$2\xi\,\omega_n = K_1 \cdot K_2,$$

$$\therefore \quad \xi = \frac{1}{2}\sqrt{K_1}\cdot K_2$$

Now, M_p is function of ξ alone,

$$\therefore \quad \%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$\text{i.e.} \quad 25 = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

$$\therefore \quad 0.25 = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

i.e. $$ln\,(0.25) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

∴ $$-1.3862 = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$ and solving $\xi = 0.4037$

Now, $$T_p = \frac{\pi}{\omega_d} = 4 \text{ sec.}$$

i.e. $$\frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 4$$

∴ $$\frac{\pi}{\omega_n\sqrt{1-(0.4037)^2}} = 4$$ i.e. $\omega_n = 0.8584$ rad/sec.

Now, $$\omega_n = \sqrt{K_1}$$

i.e. $$K_1 = \omega_n^2 = 0.7369$$

and, $$\xi = \frac{1}{2}\sqrt{K_1}\cdot K_2 = 0.4037$$

i.e. $$K_2 = 0.9405$$

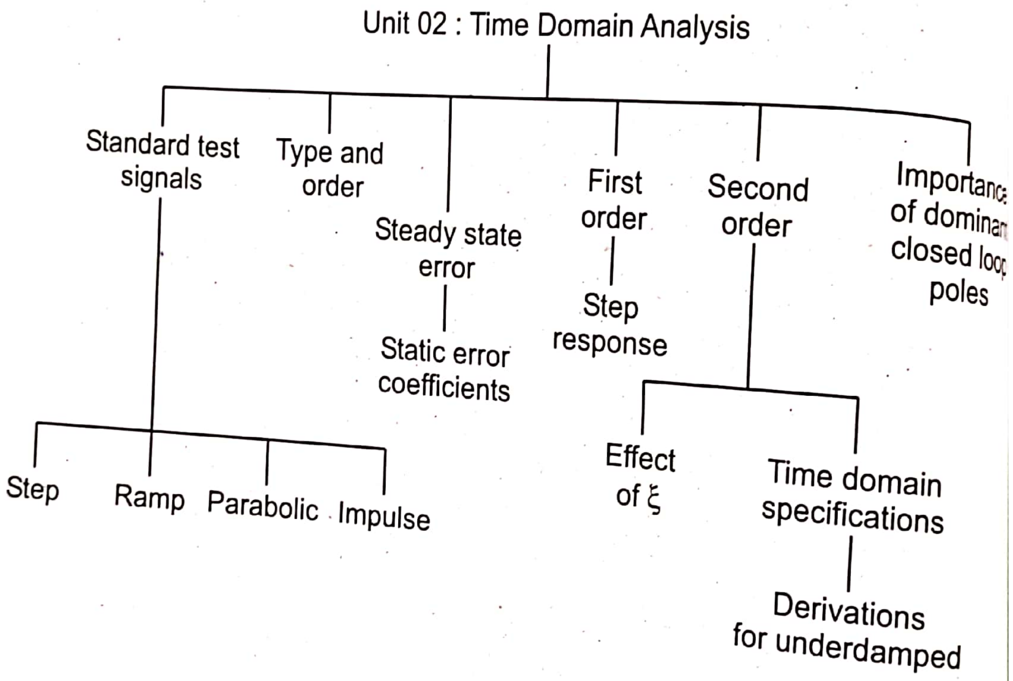
2.12 : Importance of Dominant Closed Loop Poles of Higher Order Systems

Important Points to Remember

- The closed loop poles which are located in the left half of s-plane very near to the imaginary axis are called dominant poles.
- All the closed loop poles which are located far away from the imaginary axis in the left half of s-plane can be neglected for higher order systems and system can be approximated as second order system intems of the dominant pair of complex poles.

- When a closed loop pole is located near the imaginary axis i left half of s-plane, then it takes longer time for correspondin exponential term in the response to vanish from transient par Thus it controls the transient behaviour of the system. Henc such closed loop poles are called dominant closed loop poles.

Memory Map


Unit 02 : Time Domain Analysis
Standard test signals
Type and order
Steady state error
First order
Second order
Importanc of dominan closed loo poles
Static error coefficients
Step response
Step
Ramp
Parabolic
Impulse
Effect of ξ
Time domain specifications
Derivations for underdamped

END...

UNIT - 3

3

Stability Analysis and Root Locus

3.1 : Concept of Stability

Important Points to Remember

- The analysis of a system whether it can reach steady state, passing through the transients successfully and whether it returns to zero position when input is removed, is called stability analysis of a system.

Q.1 Define : i) Stable system ii) Unstable system iii) Critically or Marginally stable system iv) Conditionally stable system

☞[SPPU : Dec.-99, 01, 02, 05, May-01, 03, 04, 07, Marks 7]

Ans. : i) A linear time invariant system is said to be **stable** if following conditions are satisfied :

a) When the system is excited by a bounded input, output is also bounded and controllable.

b) In the absence of the input, output must tend to zero irrespective of the initial conditions.

- This is called bounded input bounded output (BIBO) stability.

ii) A linear time invariant system is said to be **unstable** if,

a) For a bounded input, it produces an unbounded output.

b) In absence of the input, output may not return to zero. It shows certain output without input.

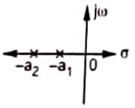
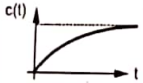
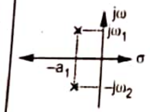
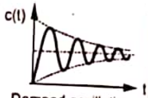
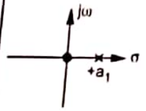
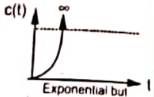
iii) A linear time invariant system is said to be **critically or marginally stable** if for a bounded input, its output oscillates with constant frequency and amplitude. Such oscillations of output are called **undamped oscillations or sustained oscillations.**

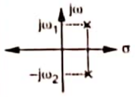
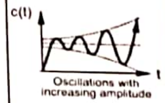
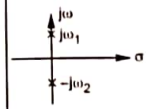
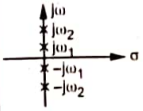
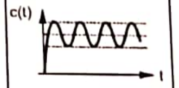
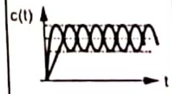
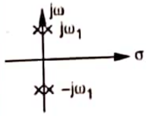
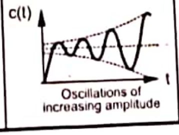
iv) A linear time invariant system is said to be **conditionally stable** if for a certain condition of a particular parameter of the system, its output is bounded one. Otherwise if that condition is violated output becomes unbounded and system becomes unstable. Thus the stability of the system depends on condition of a particular parameter of the system. Such a system is called **conditionally stable system.**

3.2 : Nature of Response for Various Locations of Roots in s-plane

Q.2 How the nature of response and stability of a system depends on the locations of roots of characteristic equation in s-plane ?

Ans. : The roots of characteristic equation of a system are its closed loop poles.

Sr. No.	Nature of closed loop poles	Locations of closed loop poles in s-plane	Step response	Stability condition
1.	Real, negative i.e. in L.H.S. of s-plane		 Pure exponential	Absolutely stable
2.	Complex conjugate with negative real part i.e. in L.H.S. of s-plane		 Damped oscillations	Absolutely stable
3.	Real, positive i.e. in R.H.S. of s-plane (Any one closed loop pole in right half irrespective of number of poles in left half of s-plane)		 Exponential but increasing towards ∞	Unstable

4.	Complex conjugate with positive real part i.e. in R.H.S. of s-plane			Unstable
5.	Non repeated pair on imaginary axis without any pole in R.H.S. of s-plane	 or  two non repeated pairs on imaginary axis.	 Frequency of oscillations = ω_1  Sustained oscillations with two frequency components ω_1 and ω_2	Marginally or critically stable Marginally or critically stable.
6.	Repeated pair on imaginary axis without any pole in R.H.S. of s-plane			Unstable

3.3 : Concept of Relative Stability

Important Points to Remember

- System is said to be relatively more stable if settling time that system is less than that of other system. It indicates h fast the system reaches to a steady state and how fast transient part of the system dies out completely.
- It is inversely proportional to the real parts of the roots of characteristic equation of the system. The roots located ne the imaginary axis in s-plane produce larger settling time. the roots move away from the imaginary axis in left half s-plane, the settling time reduces making the system relative more stable.
- Hence the roots of the characteristic equation i.e. closed loo poles located near the imaginary axis in s-plane are calle **dominant roots or poles**, which decide the stability of th system.

3.4 : Routh-Hurwitz Criterion

Q.3 What is stability ? Give the necessary conditions for stability.

☞ [SPPU : May-17, Marks 4]

Ans. : For stability, refer answer of Q.1.

- In order that the characteristic equation of a system has no roo in right of s-plane, it is **necessary but not sufficient** that,

1) All the coefficients of the polynomial have the same sign.
2) None of the coefficient vanishes i.e. all powers of 's' must b present in descending order from 'n' to zero.

Q.4 State and explain Routh's stability criterion.

☞ [SPPU : Dec.-01, 07, 15, 16, 17, May-2000, 03, 05, 09, 15, 16, Marks 6]

Ans. : Consider the general characteristic equation as,

$$F(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots\ldots + a_n = 0$$

- The Routh-Hurwitz array is then obtained as,

$$\begin{array}{l|lllll}
s^n & a_0 & a_2 & a_4 & a_6 & \ldots\ldots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \\
s^{n-2} & b_1 & b_2 & b_3 & & \\
s^{n-3} & c_1 & c_2 & c_3 & & \\
: & : & : & : & & \\
s^0 & a_n & & & &
\end{array}$$

- Coefficients for first two rows are written directly from characteristic equation.

From these two rows, next rows can be obtained as follows.

$$b_1 = \frac{a_1\,a_2 - a_0\,a_3}{a_1},\ b_2 = \frac{a_1\,a_4 - a_0\,a_5}{a_1},\ b_3 = \frac{a_1\,a_6 - a_0 a_7}{a_1}$$

- From 2nd and 3rd row, 4th row can be obtained as

$$c_1 = \frac{b_1\,a_3 - a_1\,b_2}{b_1},\ \ c_2 = \frac{b_1\,a_5 - a_1\,b_3}{b_1}$$

- This process is to be continued till the coefficient for s^0 is obtained which will be a_n. From this array stability of a system can be predicted.

- The necessary and sufficient condition for system to be stable is,

> "All the terms in the first column of Routh's array must have same sign. There should not be any sign change in the first column of Routh's array."

- If there are any sign changes existing then,

a) System is unstable.

b) The number of sign changes equals the number of roots lying in the right half of the s-plane.

Q.5 Using Routh's Hurwitz criterion for stability analysis determine stability of following system P(s) = $s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$. From that determine how many roots are there in left half s plane, right half s plane, and on imaginary axis. ☞ [SPPU : Dec.-17, Marks 8]

- The Routh-Hurwitz array is then obtained as,

$$\begin{array}{l|lllll}
s^n & a_0 & a_2 & a_4 & a_6 & \ldots\ldots \\
s^{n-1} & a_1 & a_3 & a_5 & a_7 & \\
s^{n-2} & b_1 & b_2 & b_3 & & \\
s^{n-3} & c_1 & c_2 & c_3 & & \\
\vdots & \vdots & \vdots & \vdots & & \\
s^0 & a_n & & & &
\end{array}$$

- Coefficients for first two rows are written directly from characteristic equation.

From these two rows, next rows can be obtained as follows.

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}, \; b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}, \; b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

- From 2nd and 3rd row, 4th row can be obtained as

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}, \; c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

- This process is to be continued till the coefficient for s^0 is obtained which will be a_n. From this array stability of a system can be predicted.
- The necessary and sufficient condition for system to be stable is,

> "All the terms in the first column of Routh's array must have same sign. There should not be any sign change in the first column of Routh's array."

- If there are any sign changes existing then,

a) System is unstable.

b) The number of sign changes equals the number of roots lying in the right half of the s-plane.

Q.5 Using Routh's Hurwitz criterion for stability analysis determine stability of following system $P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$. From that determine how many roots are there in left half s plane, right half s plane, and on imaginary axis. ☞ **[SPPU : Dec.-17, Marks 8]**

Ans. : For given equation, Routh's array is,

$$\begin{array}{c|ccc} s^5 & 1 & 5 & 1 \\ s^4 & 3 & 4 & 3 \\ s^3 & 3.667 & 0 & 0 \\ s^2 & 4 & 3 & \\ & \downarrow & & \\ s^1 & -2.75 & 0 & \\ & \downarrow & & \\ s^0 & 3 & & \end{array}$$

There are two sign changes in first colum without special case.

$\therefore$ Roots in right half of s-plane = **2.**

$\therefore$ Roots on imaginary axis = **0.**

$\therefore$ Roots in left half of s-plane = 5 – 2 = 3 (As n = 5 = total roots)

Hence the system is **unstable.**

3.5 : Special Cases of Routh's Array

Q.6 Which are the two special cases of Routh's array ? How these cases can be handled ? ☞ **[SPPU : Dec.-15, April-18 (In sem), Marks 4]**

Ans. : There are two special cases of Routh's array.

Special case 1 : The appearance of first element of any row as zero while same remaining row consisting of at least one nonzero element.

- In such a case, that zero is replaced by a small positive number + ε and array is completed interms of ε.
- The signs of the first column elements are examined by taking Lim ε → 0 and the stability is predicted.

Special case 2 : There occurs a complete row as row of zeros while generating an array. So array terminates abruptly as shown,

$$\begin{array}{c|ccc} s^n & a_1 & a_2 & a_3 \\ s^{n-1} & b_1 & b_2 & b_3 \\ s^{n-2} & c_1 & c_2 & c_3 \\ s^{n-3} & 0 & 0 & 0 \end{array} \quad \Leftarrow \text{Row of zeros}$$

- In such case an equation is formed using the coefficients of a row which is just above the row of zeros. Such an equation is called an **Auxiliary Equation,** denoted as A(s). The auxiliary equation is always odd or even polynomial in s.

$$A(s) = c_1 \; s^{n-2} + c_2 \; s^{n-4} + c_3 \; s^{n-6} \; \ldots\ldots$$

- Differentiate this with respect to s.

$$\frac{d\,A(s)}{d\,s} = c_1\,(n-2)\; s^{n-3} + c_2\,(n-4)\; s^{n-5} + c_3\,(n-6)\; s^{n-7} \; \ldots\ldots$$

- Complete the array, replacing row of zeros by the coefficients of the equation $dA(s)/ds$.
- If at all, there is any sign change in the first column of the completed array then the given system is unstable.
- But if there is no sign change in the first column then it is confirmed that there is no closed loop pole in right half of s-plane. But system may be stable and its stability is determined by solving the equation $A(s)=0$ i.e. auxiliary equation for its roots. From the locations of these roots it must be decided whether system is stable or not.

> Remember that the roots of $A(s)=0$ are the **dominant roots** of $F(s) = 0$ and stability is totally dependent on the locations of roots of $A(s)=0$ in such special case.

Q.7 Comment on stability using Routh criteria if characteristic equation is : Q(s) = $s^5 + 2s^4 + 3s^3 + 4s^2 + 5s + 6 = 0$ How many poles lie in right half of s-plane ? ☞ [SPPU : Dec.-14, Marks 4]

Ans. : The Routh's array is,

$$\begin{array}{c|ccc}
s^5 & 1 & 3 & 5 \\
s^4 & 2 & 4 & 6 \\
s^3 & 1 & 2 & 0 \\
s^2 & 0 & 6 & \\
 & +\varepsilon & & \\
s^1 & \dfrac{2\varepsilon-6}{\varepsilon} & 0 & \\
s^0 & 6 & &
\end{array}
\qquad
\begin{array}{l}
\\ \\ \\
\Leftarrow \text{Special case 1} \\
+\varepsilon \text{ is +ve} \\
\lim\limits_{\varepsilon \to 0} \dfrac{2\varepsilon-6}{\varepsilon} \to -\text{Ve} \\
\\
\end{array}$$

> Thus there are two sign changes in the first column hence two poles lie in right half of s-plane.

Q.8 Construct Routh array and determine the stability of the system whose characteristic equation is $s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. ☞ **[SPPU : May-18, Marks 7]**

Ans. : The Routh's array is,

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0
	8	24	0	
s^2	6	16	0	
s^1	2.667	0		
s^0	16			

$\Longrightarrow$ Special case 2

$$A(s) = 2s^4 + 12s^2 + 16$$

$$\therefore \frac{dA(s)}{ds} = 8s^3 + 24s$$

There is no sign change in the first column of completed array. But due to special case 2, system **may be** stable.

To obtain stability, solve $A(s) = 0$

i.e. $2s^4 + 12s^2 + 16 = 0$

$\therefore$ $s^2 = -2, -4$

i.e. $s = \pm j\sqrt{2}, \pm j2$

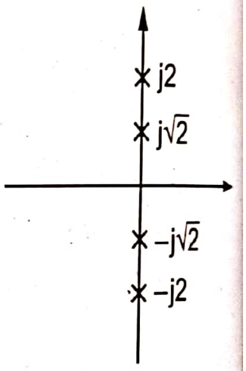


Fig. Q.8.1

As the roots of A(s) = 0 are complex conjugates, purely imaginary and non-repeated, the system is **marginally stable in nature.**

Q.9 Using Routh Hurwitz criterion determine whether the given close loop system is unstable. If unstable how many poles are on right half s plane or imaginary axis.

$$G(S) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

☞ **[SPPU : May-16, Marks 8]**

Ans. : Given system is closed loop system hence characteristic equation is $s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56 = 0$

Routh's array is,

$$\begin{array}{c|ccc} s^5 & 1 & 6 & 8 \\ s^4 & 7 & 42 & 56 \\ s^3 & \boxed{0 \qquad 0} & & \\ & 28 & 84 & 0 \\ s^2 & 21 & 56 & \\ s^1 & 9.333 & 0 & \\ s^0 & 56 & & \end{array} \Leftarrow$$

Special case 2

$A(s) = 7s^4 + 42s^2 + 56$

$\dfrac{dA(s)}{ds} = 28s^3 + 84s$

No sign change in the first column hence no root is right half of s-plane. But stability is to be determined by solving A(s) = 0.

$\therefore \quad 7s^4 + 42\,s^2 + 56 = 0 \quad \text{i.e.} \quad s^2 + 6s^2 + 8 = 0$

$\therefore \quad (s^2 + 2)\,(s^2 + 4) = 0 \quad \text{i.e.} \quad s^2 = -\,2,\ s^2 = -\,4$

$\therefore \quad s = \pm j\sqrt{2},\ \pm j2$

As four out of 5 roots are located on imaginary axis and are nonrepeated and no root in right half of s-plane, system is **marginally stable** in nature.

3.6 : Marginal K and Frequency of Sustained Oscillations

Important Points to Remember

- Practically gain K of the forward path is unknown. The range of K for stability can be obtained by constructing Routh's array interms of K and then finding the range of K which will not create any sign change in the first column.
- The value of K which makes the system marginally stable is called marginal value of K denoted as K_{mar}. This is the value of K which makes any row other than s^0 as row of zeros in the Routh's array.
- To obtain the frequency of oscillations, solve the auxiliary equation A(s) = 0 for $K = K_{mar}$.
- The magnitude of imaginary roots of A(s) = 0 obtained for marginal value of $K(K_{mar})$ indicates the frequency of sustained oscillations, which system will produce under marginally stable condition.

Q.10 Determine the range of K for stability of a system having characteristic equation $s^3 + 3Ks^2 + (K+2)s + 4 = 0$. Find the value of K so that the system is marginally stable and find the frequency of sustained oscillations.

☞ [SPPU : April-18 (In sem), Marks 6]

Ans. : From given characteristic equation, Routh's array is,

$$\begin{array}{c|cc} s^3 & 1 & K+2 \\ s^2 & 3K & 4 \\ s^1 & \dfrac{3K^2+6K-4}{3K} & 0 \\ s^0 & 4 & \end{array}$$

From s^2 row,

$3K > 0$ is $K > 0$

From s^1 now,

$3K^2 + 6K - 4 > 0$

For marginal stability, $3K^2 + 6K - 4 = 0$

$\therefore \quad (K + 2.527)(K - 0.5275) = 0$

$\therefore \ K_{mar} = +\mathbf{0.5275}$ ... only +ve value

$A(s) = 3Ks^2 + 4 = 0$ i.e. $3 \times 0.5275\, s^2 + 4 = 0$

$\therefore \ s^2 = -2.5276$ i.e. $s = \pm j1.59$

$\therefore$ Frequency of sustained oscillations = **1.59 rad/sec**

Q.11 Using Routh Hurwitz criterion for the unity feedback control system with open loop transfer function $G(s) = \dfrac{K(s+13)}{s(s+3)(s+7)}$

i) Find range of values of K for the system to be stable

ii) Find the value of K for marginally stable system and corresponding close loop poles and frequency of sustained oscillations.

☞ [SPPU : May-17, Marks 8]

Ans. : The characteristic equation is,

$$1 + G(s)H(s) = 0 \quad \text{i.e.} \quad 1 + \frac{K(s+13)}{s(s+3)(s+7)} = 0$$

$$\therefore \quad s^3 + 10s^2 + (21+K)s + 13K = 0$$

Routh's array,

$$\begin{array}{c|cc} s^3 & 1 & 21+K \\ s^2 & 10 & 13K \\ s^1 & \dfrac{210-3K}{10} & 0 \\ s^0 & 13K & \end{array}$$

From s^0 row, $K > 0$

From s^1 row,

$210 - 3K > 0$

$\therefore\ K < 70$

i) $\boxed{0 < K < 70}$ is range of K for stability

ii) $210 - 3K = 0$ i.e. $K_{max} = \mathbf{70}$

$A(s) = 10s^2 + 13K = 0$ i.e. $10s^2 + 13 \times 70 = 0$

$\therefore\ s^2 = -91,\ s = \pm\, j\ \mathbf{9.539}$

Hence, frequency of sustained oscillations is **9.539 rad/sec.**

The closed loop poles are at $\pm\, j$ **9.539** and **– 10.**

Q.12 The open-loop transfer function of a unity feedback system is given by

$$G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$$

By applying the Routh criterion discuss the stability of the closed loop system as a function of K. Determine the value of K which will cause sustained oscillation in the closed loop system. What are the corresponding oscillation frequencies ?

☞ [SPPU : Dec.-16, Marks 7]

Ans. : The characteristic equation is $1 + G(s)H(s) = 0$, $H(s) = 1$.

$$\therefore \quad 1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

i.e. $s^4 + 12s^3 + 69s^2 + 198s + (200+K) = 0$

The Routh's array is,

$$\begin{array}{c|ccc} s^4 & 1 & 69 & 200+K \\ s^3 & 12 & 198 & 0 \\ s^2 & 52.5 & 200+K & \\ s^1 & \dfrac{10395-12(200+K)}{52.5} & 0 & \\ s^0 & 200+K & & \end{array}$$

From row of s^0,

$200 + K > 0$ i.e. $K > -200$

From row of s^1,

$10395 - 12\,(200 + K) > 0$

$\therefore\ 7995 - 12\,K > 0$

$\therefore\ K < \dfrac{7995}{12} < 666.25$

For stability, the range of values of K is **$-200 < K < 666.25$.**

$$K_{mar} = 666.25$$

...(making row of s^1 as row of zero)

... for which system is marginally stable

The auxiliary equation is, $A(s) = 52.5s^2 + 200 + K_{mar} = 0$

$$\therefore \quad s^2 = -\frac{(200+666.25)}{52.5} = -16.5 \quad \text{i.e.} \quad s = \pm j\,4.062$$

The frequency of oscillations is **4.062 rad/sec.**

3.7 : Root Locus

Q.13 Define root locus. State the angle and magnitude condition of root locus. Mention the use of these conditions.

☞ **[SPPU : May-15, Marks 3]**

Ans. : • The locus of the closed loop poles of a system, obtained when system gain 'K' is varied from 0 to + ∞ is called **Root Locus.**

- Angle condition can be stated as, $\angle G(s)H(s)|$ for any value of 's' which is the root of equation $[1 + G(s)H(s) = 0]$ is $= \pm(2q+1)\,180°$ where $q = 0, 1, 2$ ....
  i.e. Odd multiple of 180°.

- Any point in s-plane which satisfies the angle condition has to be on the root locus of the corresponding system.

- Magnitude condition is stated as,

$$\left| G(s)H(s) \right|_{\substack{\text{at a point in s - plane} \\ \text{which is on root locus}}} = 1$$

- Magnitude condition is used to find K for any point on the root locus. But the magnitude condition can be used only when it is sure that a point is on the root locus.
- Hence angle condition is used first to confirm whether a point is on the root locus or not and then once confirmed it is followed by the magnitude condition to find the corresponding calue of K.

3.8 : Rules to Construct Root Locus

Q.14 Explain the rules for the construction of root locus of a system. ☞ [SPPU : May-15, 16, 17, Dec.-15, 17, Marks 8]

OR Explain centroid and angles of asymptotes of a root locus.

☞ [SPPU : April-18 (In sem), Marks 8]

Ans. : • The various rules to construct the root locus are,

Rule 1 : The root locus is always symmetrical about the real axis.

Rule 2 : If G(s)H(s) = Open loop T.F. of the system and

P = Number of open loop poles,

Z = Number of open loop zeros then

Number of branches equal to number of open loop poles (N = P).

- Branches will start from each of the location of open loop pole. Out of 'P' number of branches, 'Z' number of branches will terminate at the locations of open loop zeros. The remaining 'P – Z' branches will approach to infinity. The branch direction always remains from open loop poles towards open loop zeros.

Rule 3 : A point on the real axis lies on the root locus if the sum of the number of open loop poles and the open loop zeros, on the real axis, to the right hand side of this point is odd.

Rule 4 : Angles of aysmptotes

- The 'P–Z' branches approach to infinity along the straight lines called **Asymptotes** of the root locus. Asymptotes are the guidelines for the branches approaching to infinity.

- Angles of such asymptotes are given by ,

$$\boxed{\theta = \frac{(2q+1)\,180^\circ}{P-Z} \quad \text{where } q = 0, 1, 2 \ldots (P-Z-1)}$$

Rule 5 : Centroid

- All the asymptotes intersect the real axis at a common po known as **centroid** denoted by σ. The co-ordinates of centroid c be calculated as,

$$\sigma \text{ (centroid)} = \frac{\sum \text{Real parts of poles of } G(s)H(s) - \sum \text{Real parts of zeros of } G(s)H(}{P - Z}$$

- Centroid is always real, it may be located on negative or positi real axis. It may or may not be the part of the root locus.

Rule 6 : **Breakaway Point**

- Breakaway point is a point on the root locus where multip roots of the characteristic equation occurs, for a particular val of K.

- Steps to determine the co-ordinates of breakaway points are,

Step 1 : Construct the characteristic equation 1 + G(s)H(s) = 0 of the system.

Step 2 : From this equation, separate the terms involving `K' and terms involving `s'. Write the value of K in terms of s i.e. K = F(s).

Step 3 : Differentiate above equation w.r.t. `s', equate it to zero

i.e. $\frac{dK}{ds} = 0$

Step 4 : Roots of the equation $\frac{dK}{ds} = 0$ gives us the breakaway points.

- Out of all the roots of $\frac{dK}{ds} = 0$, those for which the value of K i positive are valid for the root locus.

- The root locus branches always leave breakaway points at an angle of $\pm\frac{180^\circ}{n}$ where n = Number of branches approaching at breakaway point.

Rule 7 : **Intersection of root locus with imaginary axis**

- This can be determined by constructing Routh's array interms of K. Find the marginal value of K. Solve $A(s) = 0$ for marginal value of K. The roots of $A(s) = 0$ for K_{mar} are the intersection points of root locus with the imaginary axis.

Rule 8 : Angle of departure at complex conjugate poles and angle of arrival at complex conjugate zeros.

- The angle of departure is the angle at which a branch departs from a complex pole denotes as ϕ_d. It is obtained as,

$$\phi_d = 180^\circ - \phi \qquad \text{where } \phi = \sum \phi_P - \sum \phi_Z$$

where $\sum \phi_P$ = Contributions by the angles made by remaining open loop poles at the pole at which ϕ_d is to be calculated.

$\sum \phi_Z$ = Contributions by the angles made by the open loop zeros at the pole at which ϕ_d is to be calculated.

- The angle of arrival is the angle at which a branch arrives at a complex zero, denotes as ϕ_a.
- Angle of arrival at a complex zero is given by,

$$\phi_a = 180^\circ + \phi \qquad \text{where } \phi = \sum \phi_P - \sum \phi_Z$$

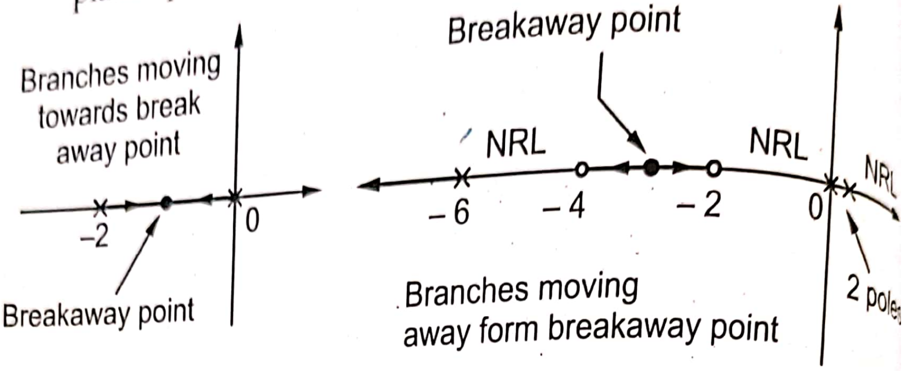
- To calculate $\Sigma\, \phi_P$, join all the remaining poles to the complex pole under consideration. Add all the angles subtended by these phasors with respect to positive x-axis. Similarly join all the zeros to pole under consideration and adding all angles subtended by these phasors with respect to positive x-axis, determine $\Sigma \phi_Z$.

Important Points to Remember

General predictions for existence of breakaway point in root locus :

1) If there are adjacently placed poles on the real axis and the real axis between them is a part of the root locus then there

exists minimum one breakaway point in between adjac placed poles. This is shown in the Fig. 3.1 (a).


Branches moving towards break away point
−2
0
Breakaway point
Breakaway point
NRL
NRL
NRL
− 6
− 4
− 2
0
Branches moving away form breakaway point
2 poles

(a) Fig. 3.1 (b)

2) If there are two adjacently placed zeros on real axis and section of real axis in between them is a part of the root locus then there exists minimum one breakaway point in between adjacently placed zeros. This is shown in the Fig. 3.1 (b).

3) If there is a zero on the real axis and to the left of that zero there is no pole or zero existing on the real axis and complete real axis to the left of this zero is a part of the root locus then there exists minimum one breakaway point to the left of that zero. This is shown in the Fig. 3.1 (c).

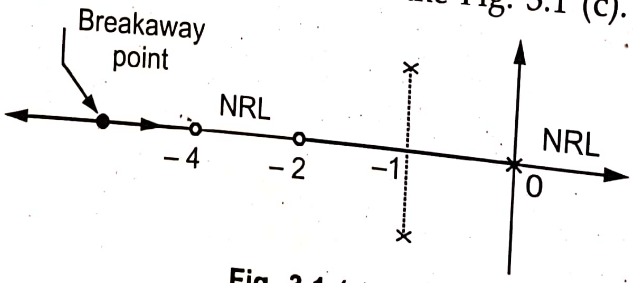

Breakaway point
NRL
NRL
− 4
− 2
−1
0

Fig. 3.1 (c)

Q.15 Determine Breakaway point and angle of departure for root locus of a system with open loop transfer function.

$$G(s)H(s) = \frac{K(s+1)}{s^2+4s+13}$$

☞ **[SPPU : April-18 (In sem), Marks 6]**

Ans. : For breakaway point,

$$1 + G(s)H(s) = 0 \quad \text{i.e.} \quad 1+\frac{K(s+1)}{s^2+4s+13} = 0$$

$$\therefore \quad s^2+4s+13+K(s+1)=0 \quad \text{i.e.} \quad K=\frac{-s^2-4s-13}{(s+1)}$$

$$\frac{dK}{ds}=0 \quad \text{i.e.} \quad (s+1)(-2s-4)-(-s^2-4s-13)(1)=0$$

Solving, $s^2+2s-9=0$

$$s = 2.162,\ -4.162$$

$\therefore$ Thus $s =$ **– 4.162** is valid breakaway point.

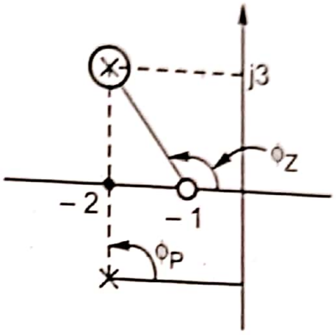


Fig. Q.15.1

For ϕ_d, consider pole at – 2 + j3.

$$\phi_Z = 180° - \tan^{-1}\frac{3}{1} = 108.43°$$

$$\phi_P = 90°$$

$$\therefore \quad \phi = \sum\phi_P - \sum\phi_Z = -18.43°$$

$$\therefore \quad \phi_d = 180° - \phi = 180° - (-18.43°) = \mathbf{198.43°} \quad \text{at } -2+j3$$

$$\phi_d = -\mathbf{198.43°} \text{ at } -2-j3$$

3.9 Steps to Construct Root Locus

Important Points to Remember

Step 1 : Get the general information about number of open loop poles, zeros, number of branches etc. from G(s)H(s).

Step 2 : Draw the pole-zero plot. Identify sections of real axis for the existence of the root locus. And predict minimum number of breakaway points by using general predictions.

Step 3 : Calculate angles of asymptotes,

$$\theta = \frac{(2\theta+1)\,180°}{P-Z} \qquad \text{where } \theta = 0,\ 1\ ,\ 2\ .....\ (P-Z-1)$$

Step 4 : Determine the centroid.

$$\sigma(\text{centroid}) = \frac{\sum \text{Real parts of poles of } G(s)H(s) - \sum \text{Real parts of zeros of } G(s)H(s)}{P-Z}$$

Step 5 : Calculate the breakaway and breakin points. If breakaway points are complex conjugates, then use angle condition to check them for their validity as breakaway points.

Step 6 : Calculate the intersection points of root locus with the imaginary axis. These are the roots of the equation A(s) = 0 for marginal value of K, obtained from Routh's array.

Step 7 : Calculate the angles of departures or arrivals for the complex conjugate poles and zeros, if present.

Step 8 : Combine steps 1 to 7 and draw the final sketch of the root locus.

Step 9 : Predict the stability and performance of the given system by using the root locus.

Q.16 How to determine the value of K for specific damping ratio for a system from its root locus ?

Ans. :

Step 1 : Draw the root locus to the scale on graph paper. Preferably choose same scale for X and Y axis.

Step 2 : Get the value of $\theta = \cos^{-1} \xi$.

Step 3 : Draw a line at angle 'θ' from origin such that 'θ' is **measured from negative real axis** in clockwise direction.

Step 4 : Determine the intersection point of this line with the root locus sketched to the scale.

Step 5 : For this point, apply the magnitude condition to decide the corresponding system gain 'K'.

Q.17 Sketch the root locus for the open-loop transfer function of a unity feedback control system given below and determine.
i) The value of K for $\xi = 0.5$
ii) The value of K for marginal stability

$$G(s) = \frac{K}{s(s+1)(s+3)}$$

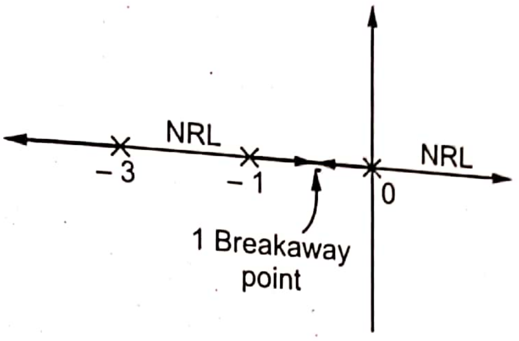
☞ **[SPPU : Dec.-04, 09, 10, 08, Marks 9]**

Ans. : Step - 1 : $P = 3,\ Z = 0,$
$N = P = 3,\ P - Z = 3$ to ∞

Starting : $s = 0,\ -1,\ -3,$

Terminating : $\infty,\ \infty,\ \infty$

Step - 2 : Real axis root locus is shown in the Fig. Q.17.1 (a).



(a)

Fig. Q.17.1

Step - 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z},\ q = 0,\ 1,\ 2.$$

$$\theta_1 = 60^\circ,\ \theta_2 = 180^\circ,\ \theta_3 = 300^\circ$$

Step - 4 :
$$\text{Centroid} = \frac{\sum \text{R.P. of O.L. poles} - \sum \text{R.P. of O.L. zeros}}{P-Z}$$

$$= \frac{0-1-3}{3} = -\mathbf{1.333.}$$

Step - 5 : Breakaway point

$$1 + G(s)H(s) = 0 \text{ gives } 1+\frac{K}{s(s+1)(s+3)} = 0$$

$$\therefore\ s^3 + 4s^2 + 3s + K = 0 \quad \text{i.e.} \quad K = -s^3 - 4s^2 - 3s \qquad \ldots(1)$$

$$\therefore\ \frac{dK}{ds} = 0 \text{ gives } 3s^2 + 8s + 3 = 0$$

Solving, $s = -0.45,\ -2.215$

Hence $s = -0.45$ is valid breakaway point.

Step - 6 : Intersection with jω axis.

$1 + G(s)H(s) = 0$, i.e. $s^3 + 4s^2 + 3s + K = 0$

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & K \\ s^1 & \frac{12-K}{4} & 0 \\ s^0 & K & \end{array}$$

From s^1 row, $12 - K = 0$

$\therefore\ K_{mar} = \mathbf{12}$

From s^2 row, $4s^2 + k = 0$

$$\therefore\ s^2 = -\frac{12}{4} = -3$$

$\therefore$

$s = \pm j\ \mathbf{1.732}$ ... Intersection with jω axis

Step - 7 : Angles of departure not required as no complex open loop poles.

Step - 8 : The root locus is shown in the Fig. Q.17.1 (b).

To find K for $\xi = 0.5$, draw a line from origin making an $\cos^{-1}\xi = 60°$ with negative real axis.

Find its intersection with root locus which is point P on root locu

From Fig. Q.17.1 (b), P is $s = -0.4 + j0.75$

Using angle condition at P,

$$|G(s)H(s)|_{\text{At P}} = 1 \quad \text{i.e.} \quad \left|\frac{K}{s(s+1)(s+3)}\right|_{s=-0.4+j0.75} = 1$$

$$\therefore \quad \frac{|K|}{|-0.4+j\,0.75|\,|-0.4+j0.75+1|\,|-0.4+j0.75+3|} = 1$$

$$\therefore \quad \frac{K}{0.85 \times 0.9604 \times 2.706} = 1 \quad \text{i.e.} \quad K = \mathbf{2.209} \text{ for } \xi = 0.5$$

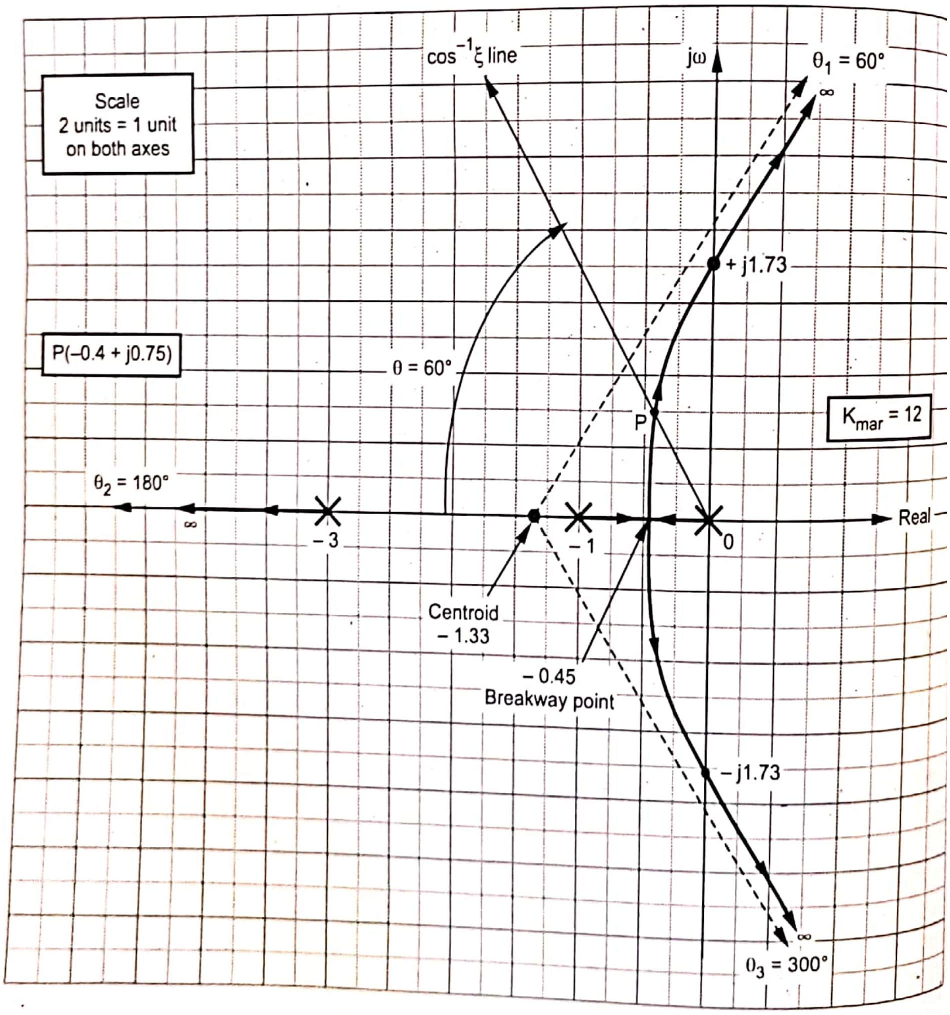


Fig. Q.17.1 (b)

Q.18 Sketch the root locus of the unity feedback system having $G(s) = \frac{K}{s(s+2)(s+4)}$ where K is varied from 0 to ∞. Hence obtain the value of K for which the system is unstable.

☞ [SPPU : Dec.-09, May-03, 06, 18, Marks 7]

Ans. : Refer Q.17 for the procedure and verify the answer as :

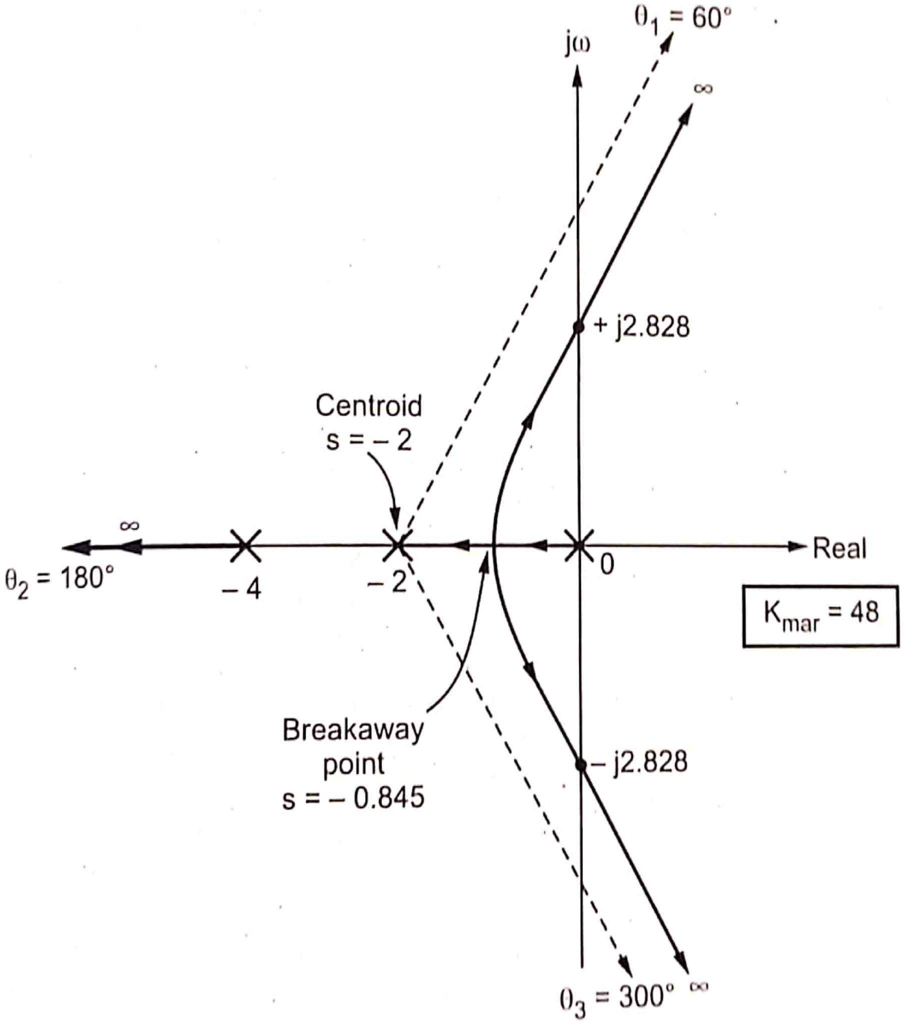

θ1 = 60°
jω
∞
+ j2.828
Centroid s = – 2
∞
Real
θ2 = 180°
– 4
– 2
0
Kmar = 48
Breakaway point s = – 0.845
– j2.828
θ3 = 300°
∞

Fig. Q.18.1

- For K > 48, the system is unstable.

Q.19 A unity feedback control system has a open loop transfer function.

$$G(s) = \frac{K}{s(s^2+4s+13)}$$

Sketch the root locus plot of the system by determining the following :

i) Centroid, number and angle of asymptotes.

ii) Angle of departure of root loci from the poles.

iii) Breakaway point if any

iv) The value of K and the frequency at which the root loci cross the jω - axis.

☞ **[SPPU : Dec.-06, 11, 16, Marks 10]**

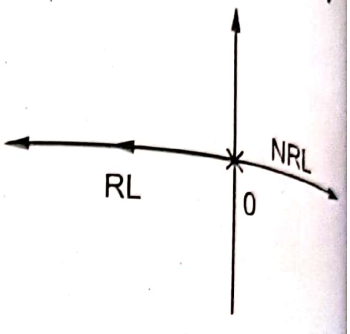
Ans. : Step - 1 : $P = 3, \; Z = 0,$

$N = P = 3, \; P - Z = 3$

3 branches approach to ∞.

Starting : $s = 0, \; -2 \pm j3$

Terminating : $s = \infty, \infty, \infty$



(a)
Fig. Q.19.1

Step - 2 : Real axis root locus.

Step - 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \quad q = 0, 1, 2$$

$$\therefore \quad \theta_1 = 60^\circ, \quad \theta_2 = 180^\circ, \; \theta_3 = 300^\circ$$

Step - 4 : Centroid $= \dfrac{\sum \text{R.P. of O.L. poles} - \sum \text{R.P. of O.L. zeros}}{P-Z}$

$$= \frac{0-2-2-0}{3} = -\mathbf{1.333}$$

Step - 5 : Breakaway point.

No breakaway point as per step 2.

Step - 6 : Intersection with jω axis.

$$1 + G(s)H(s) = 0 \quad \text{i.e.} \quad 1 + \frac{K}{s(s^2+4s+13)} = 0$$

$$\therefore \; s^3 + 4s^2 + 13s + K = 0$$

$$\begin{array}{c|cc} s^3 & 1 & 13 \\ s^2 & 4 & K \\ s^1 & \frac{52-K}{4} & 0 \\ s^0 & K & \end{array}$$

From s^1 row,

$52 - K = 0 \qquad \therefore \; K_{mar} = 52$

From s^2 row,

$4s^2 + 52 = 0$

$$\therefore \; s^2 = -13, \qquad s = \pm \, \mathbf{j3.605}$$

Step - 7 : Angle of departure

Consider $-2+j3$ as shown in the Fig. Q.19.1 (b)

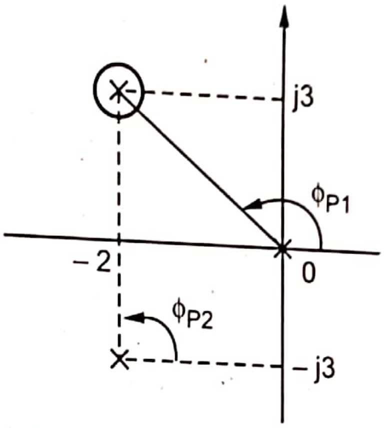
$$\therefore \phi_{P1} = 180° - \tan^{-1}\frac{3}{2} = 123.69°, \ \phi_{P2} = 90°$$

$$\sum \phi_P = 123.69° + 90° = 213.69° \quad \sum \phi_Z = 0°$$

$$\therefore \quad \phi = \sum \phi_P - \sum \phi_Z = 213.69°$$

$$\therefore \ \phi_d = 180° - \phi = -33.69° \text{ at } -2 + j3$$

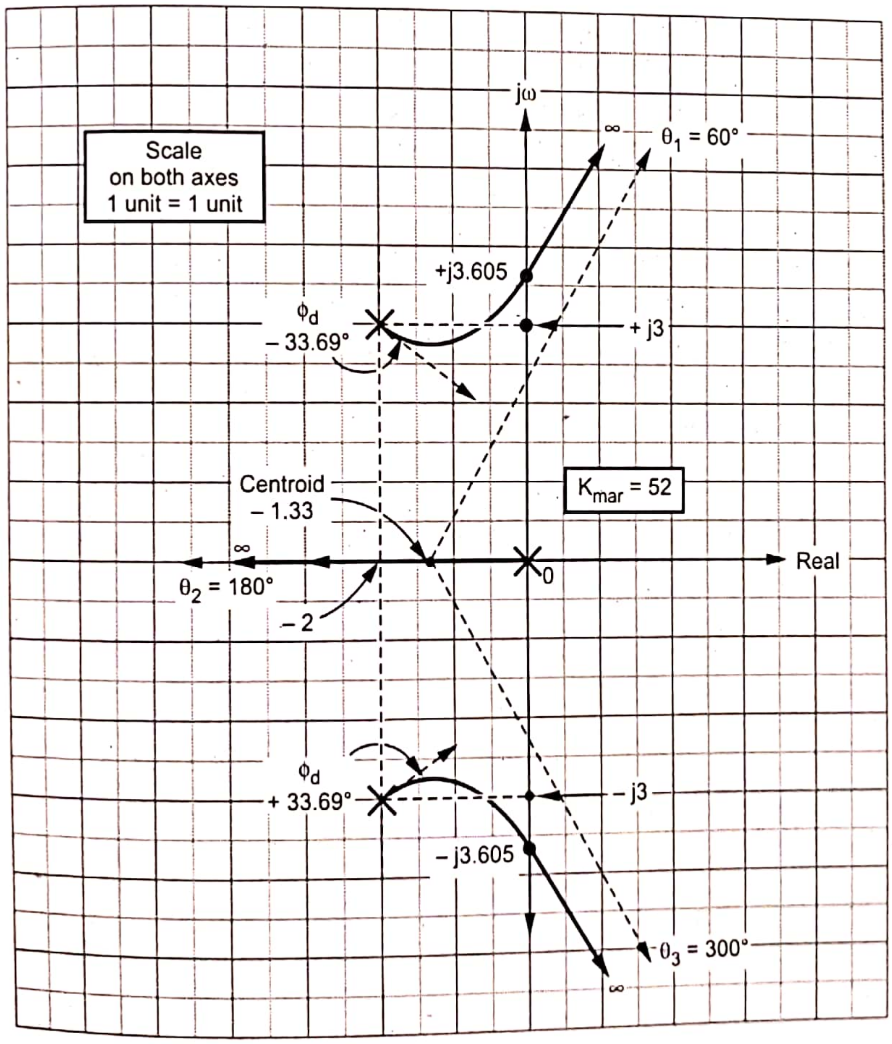
$$\phi_d = +33.69° \text{ at } -2 - j3$$



j3
ϕP1
– 2
0
ϕP2
– j3

(b)
Fig. Q.19.1

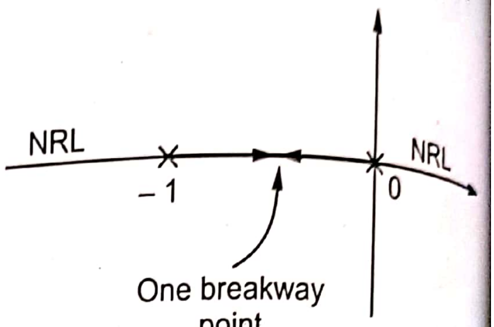
Step - 8 : Complete root locus is shown in the Fig. Q.19.1 (c).



jω
∞
θ1 = 60°
Scale
on both axes
1 unit = 1 unit
+j3.605
ϕd
– 33.69°
+ j3
Centroid
– 1.33
Kmar = 52
∞
Real
0
θ2 = 180°
– 2
ϕd
+ 33.69°
j3
– j3.605
θ3 = 300°
∞

(c) Fig. Q.19.1

Q.20 Explain for unity feedback system with open loop transfer function given as $G(s) = \dfrac{K}{s(s+1)(s^2+4s+13)}$. Draw root locus when K is varied from 0 to ∞. Also find range of values of K for which system is stable. ☞ [SPPU : Dec.-17, May-11,17, Marks 12]



(a)
Fig. Q.20.1

Ans. : **Step - 1 :** P = 4, Z = 0,
N = P = 4, P − Z = 4 → ∞
Start : s = 0, −1, −2 ± j3,
Terminating : s = ∞, ∞, ∞, ∞

Step - 2 : Section of real axis as shown in the Fig. Q.20.1 (a).

Step - 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180^\circ}{P-Z}, \; q = 0, 1, 2, 3$$

$$\theta_1 = 45^\circ, \theta_2 = 135^\circ, \theta_3 = 225^\circ, \theta_4 = 315^\circ$$

Step - 4 : Centroid $= \dfrac{0-1-2-2-0}{4} = \mathbf{-1.25}$

Step - 5 : Breakaway point

$$1 + G(s)H(s) = 0 \text{ gives } s^4+5s^3+17s^2+13s+K = 0 \quad \text{... (1)}$$

$$\therefore \quad K = -s^4-5s^3-17s^2-13s \quad \text{... (2)}$$

$$\frac{dK}{dS} = 0 \text{ gives } 4s^3+15s^2+34s+13 = 0$$

Solving, $s = -0.466, \quad -1.64 \pm j2.067$

Thus $s = \mathbf{-0.466}$ is valid breakaway point with

$K = +2.825$ from equation (2) at this point.

Step - 6 : Intersection with jω axis.

Routh's array from equation (1) is,

s^4	1	17	K
s^3	5	13	0
s^2	14.4	K	0
s^1	$\dfrac{187.2-5K}{14.4}$	0	
s^0	K		

From s^1 row, $187.2 - 5K = 0$

$\therefore \; K_{mar} = \mathbf{37.44}$

From s^2 row,

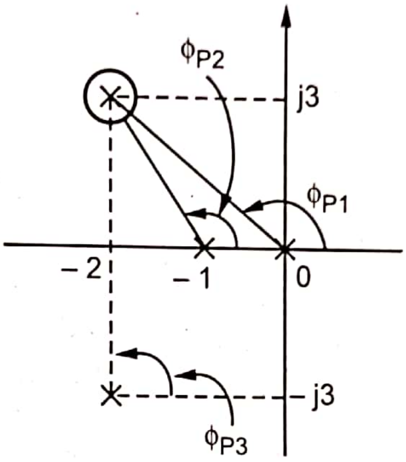
$A(s) = 14.4s^2 + K = 0$

$\therefore \; s^2 = -\dfrac{37.44}{14.4} = -2.6$

$\therefore \; s = \pm \mathbf{j1.61}$

Step - 7 : Angle of departure at complex poles.

Consider (– 2 + j2) as shown in the Fig. Q.20.1 (b)



ϕ_{P2}
j3
ϕ_{P1}
– 2
– 1
0
– j3
ϕ_{P3}

Fig. Q.20.1 (b)

$$\phi_{P1} = 180° - \tan^{-1}\frac{3}{2} = 123.69°$$

$$\phi_{P2} = 180° - \tan^{-1}\frac{3}{1} = 108.43°$$

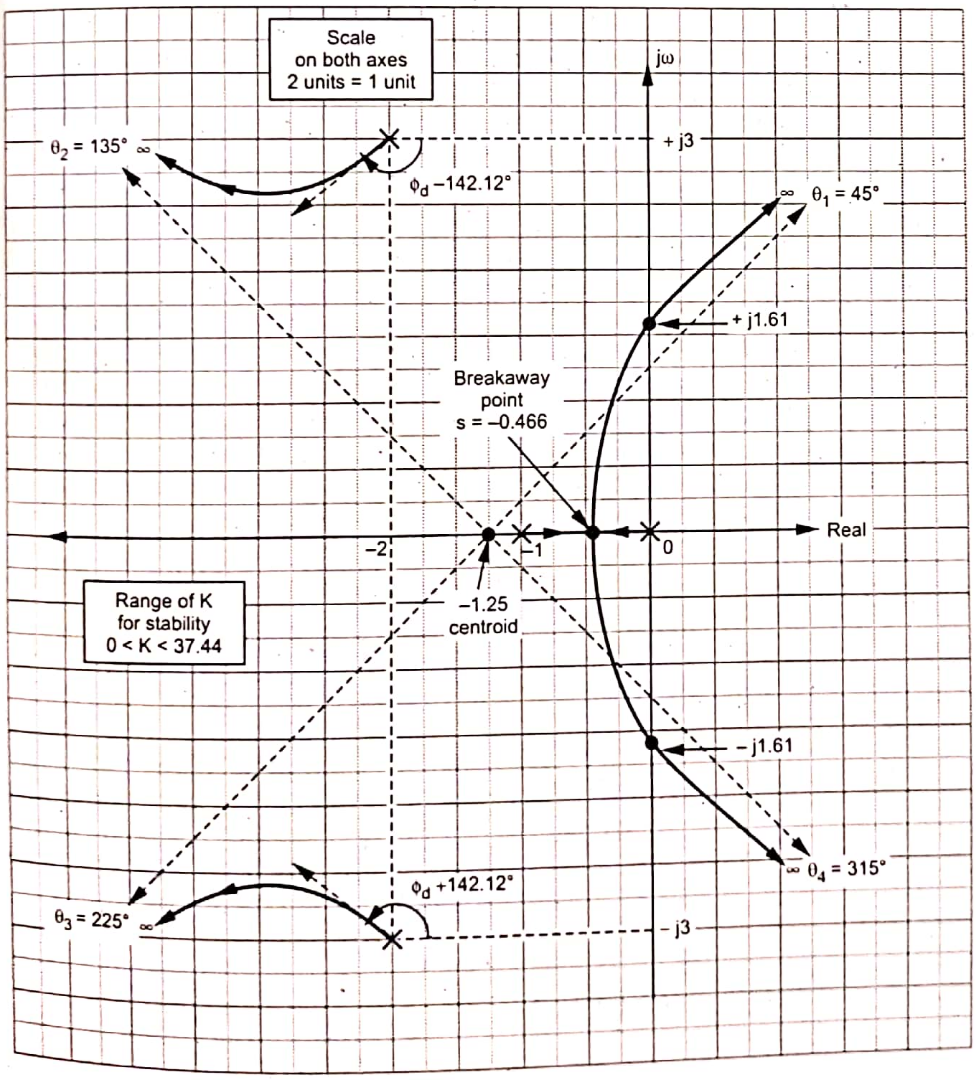
$$\phi_{P3} = 90°, \quad \sum\phi_Z = 0°$$

$$\sum\phi_P = \phi_{P1} + \phi_{P2} + \phi_{P3} = 322.12°$$

$$\therefore \quad \phi = \sum\phi_P - \sum\phi_Z = 322.12°$$

$$\therefore \quad \phi_d = 180° - \phi = -\mathbf{142.12°} \text{ at } -2 + j3 \text{ and } +\mathbf{142.12°} \text{ at } -2 - j3$$

Step-8 : The root locus is shown in the Fig. Q.20.1 (c).



Scale on both axes 2 units = 1 unit
jω
+ j3
θ_2 = 135° ∞
ϕ_d –142.12°
∞ θ_1 = 45°
+ j1.61
Breakaway point s = –0.466
Real
–2
–1
0
–1.25 centroid
Range of K for stability 0 < K < 37.44
– j1.61
∞ θ_4 = 315°
ϕ_d +142.12°
θ_3 = 225° ∞
– j3

Fig. Q.20.1 (c)

Q.21 Draw root locus for following system. Also find range of values of K for which system is stable. $G(S) = \frac{K}{s(s+2)(s^2+s+1)}$

☞ **[SPPU : May-16, Marks 10]**

Ans. : Refer Q.20 for the procedure and verify the answer as :

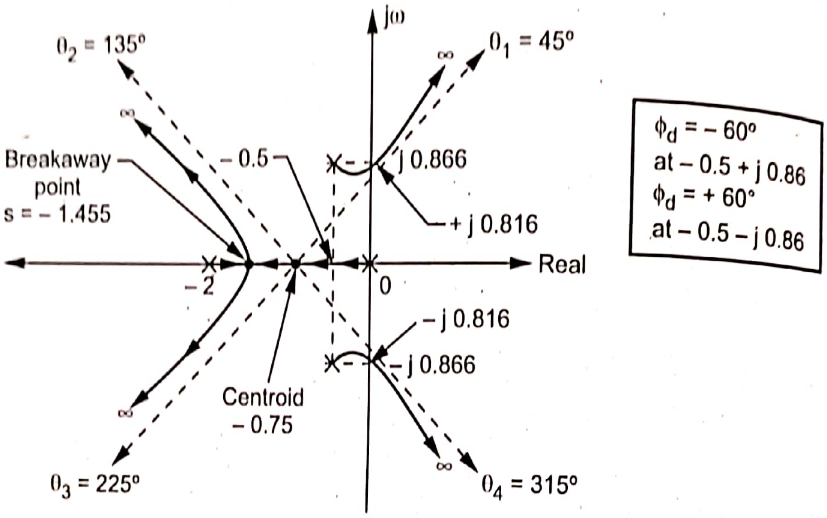

jω
θ₂ = 135°
θ₁ = 45°
∞
∞
Breakaway point s = – 1.455
– 0.5
j 0.866
+ j 0.816
Real
– 2
0
– j 0.816
– j 0.866
Centroid – 0.75
∞
∞
θ₃ = 225°
θ₄ = 315°
ϕd = – 60°
at – 0.5 + j 0.86
ϕd = + 60°
at – 0.5 – j 0.86

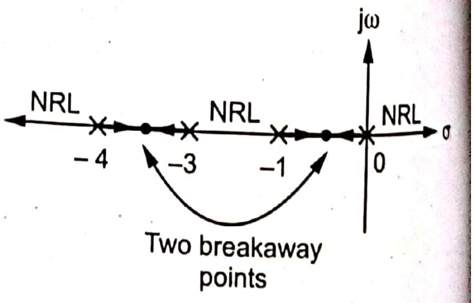
Fig. Q.21.1

Q.22 A unity feedback system has the loop transfer function.

$$G(s) = \frac{K}{s\,(s+1)\;(s+3)\;(s+4)}$$

Plot root loci by determining breakaway points and intersection with imaginary axis.

☞ **[SPPU : Dec.-12, Marks 12]**


jω
NRL
NRL
NRL
σ
– 4
–3
–1
0
Two breakaway points

Ans. : Step 1 : P = 4, Z = 0,

N = 4, P – Z = 4 branches to ∞

Starting points : s = 0, –1, – 3, – 4

Terminating points : ∞, ∞, ∞, ∞

Step 2 : Sections of real axis. Two breakaway points possible

Step 3 : Angles of asymptotes, $\theta_1 = 45°$, $\theta_2 = 135°$, $\theta_3 = 225°$, $\theta_4 = 315°$

Step 4 : Centroid $= \frac{0-1-3-4}{4} = -2$

Step 5 : Breakaway points

$1 + G(s)H(s) = 0$

i.e. $s^4 + 8s^3 + 19s^2 + 12s + K = 0$

$$\therefore \quad K = -s^4 - 8s^3 - 19s^2 - 12s$$

$$\therefore \quad \frac{dK}{ds} = -4s^3 - 24s^2 - 38s - 12 = 0 \quad \text{i.e.}$$

$$s^3 + 6s^2 + 9.5\,s + 3 = 0$$

$$\therefore \quad s = -0.4188, \quad -3.5811 \; \ldots \text{ Breakaway points}$$

Step 6 : Intersection with jω axis.

$$\begin{array}{c|ccc} s^4 & 1 & 19 & K \\ s^3 & 8 & 12 & 0 \\ s^2 & 17.5 & K & \\ s^1 & \dfrac{210-8\,K}{17.5} & 0 & \\ s^0 & K & & \end{array}$$

$$\therefore \quad 210 - 8\,K = 0$$

$$\therefore \quad K_{mar} = \frac{210}{8} = \mathbf{26.25}$$

$$A(s) = 17.5\,s^2 + K_{mar} = 0$$

$$\therefore \quad s^2 = -\frac{26.25}{17.5} \quad \text{i.e.} \quad s = \pm j\,1.224$$

Step 7 : The nature of root locus is shown in the Fig. Q.22.1.

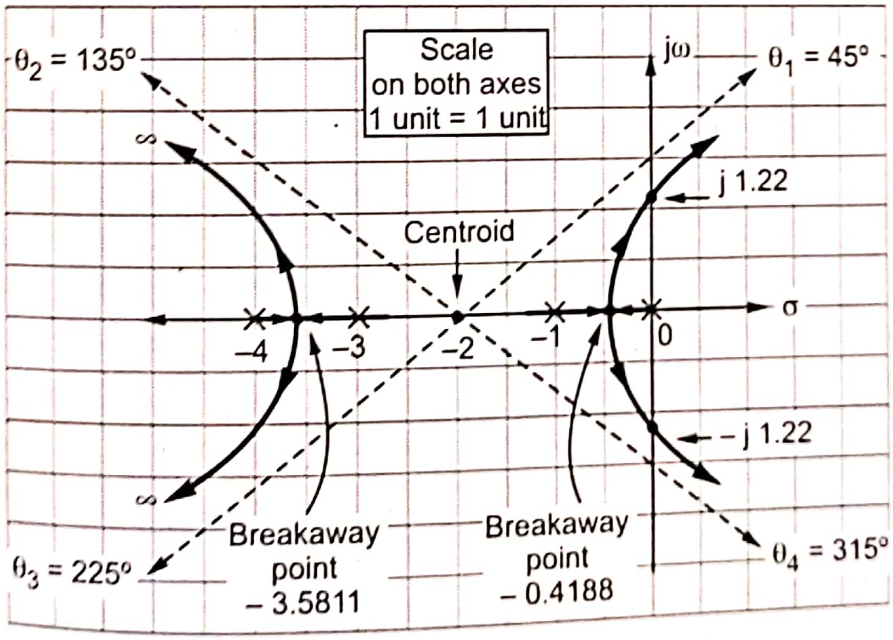
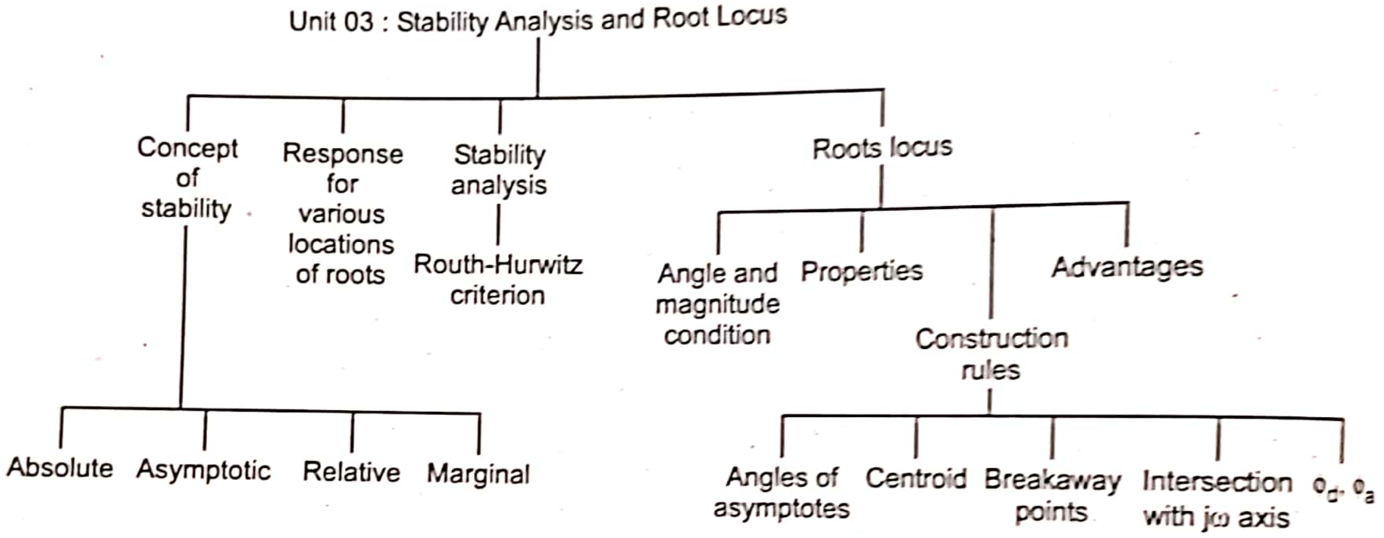

θ2 = 135°
Scale
on both axes
1 unit = 1 unit
jω
θ1 = 45°
∞
j 1.22
Centroid
σ
–4
–3
–2
–1
0
– j 1.22
∞
Breakaway
point
– 3.5811
Breakaway
point
– 0.4188
θ3 = 225°
θ4 = 315°

Fig. Q.22.1

Memory Map


Unit 03 : Stability Analysis and Root Locus
Concept of stability
Response for various locations of roots
Stability analysis
Routh-Hurwitz criterion
Roots locus
Angle and magnitude condition
Properties
Construction rules
Advantages
Absolute
Asymptotic
Relative
Marginal
Angles of asymptotes
Centroid
Breakaway points
Intersection with jω axis
ϕ_d, ϕ_a

END...

UNIT - 4

4 Frequency Domain Analysis - I

4.1 : Introduction

Q.1 What do you mean by frequency response of a system ? How to obtain frequency domain transfer function ?

Ans. : • The steady state response of a system to a purely sinusoidal input when input frequency is varied from 0 to ∞ is defined as frequency response of a system.

- In such method, the effect of change in input frequency of the input signal is studied on the magnitude and phase of the system.
- Consider a linear time invariant system with its transfer function $G(s) = \frac{C(s)}{R(s)}$. While the input is purely sinusoidal given by

$r(t) = A \sin \omega t$ hence its Laplace transform is $R(s) = \frac{A\omega}{s^2 + \omega^2}$.

$$\therefore \text{ Output } C(s) = R(s) \cdot G(s) = \frac{A\omega}{s^2 + \omega^2} \frac{N(s)}{(s+s_1)(s+s_2)\ldots.(s+s_n)}$$

- All the poles of G(s) are assumed to be located in left half of s-plane.

$$C(s) = \frac{a}{s+j\omega} + \frac{\bar{a}}{s-j\omega} + \frac{b_1}{s+s_1} + \ldots. + \frac{b_n}{s+s_n} \quad \text{.....partial fractions}$$

- Steady state of the output C_{ss} is $\lim_{t \to \infty} c(t)$. Then all the terms from c(t) except first two tems will vanish after taking inverse Laplace transform.

$$\therefore \quad c_{ss}(t) = a e^{-j\omega t} + \bar{a} e^{+j\omega t} \quad \ldots(1)$$

$$a = G(s) \cdot \frac{A\,\omega}{s^2+\omega^2} \cdot (s+j\omega)\,\Big|_{s=-j\omega} = -\frac{A\,G(-j\omega)}{2j}$$

$$\bar{a} = G(s)\,\frac{A\,\omega}{s^2+\omega^2} \cdot (s-j\,\omega)\,\Big|_{s=+j\omega} = \frac{A\,G(+j\,\omega)}{2j}$$

- $G(j\omega)$ is a complex quantity and can be expressed as
  $G(j\omega) = |G(j\omega)|\,e^{j\phi}$

where $|G(j\omega)| = \sqrt{(\text{Real part})^2 + (\text{Imj}\cdot\text{part})^2}$

and $\phi = \angle G(j\,\omega) = \tan^{-1}\left[\dfrac{\text{Imaginary part of } G(j\omega)}{\text{Real part of } G(j\omega)}\right]$

Similarly $G(-j\omega) = |G(-j\omega)|\,e^{-j\phi} = |G(j\omega)|\,e^{-j\phi}$ and use in equation (1),

$$\therefore\ c_{ss}(t) = \frac{-A\,|G(-j\omega)|}{2j}\,e^{-j\phi}\cdot e^{-j\omega t} + \frac{A\,|G(j\omega)|}{2j}\,e^{j\phi}\cdot e^{j\omega t}$$

$$= A\,|G(j\omega)|\,\frac{e^{j(\omega t+\phi)} - e^{-j(\omega t+\phi)}}{2j} \quad \text{.....}\ \sin\theta = \frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$\therefore \quad \boxed{C_{ss}(t) = A\,|G(j\omega)|\sin(\omega t+\phi)}$$

- Thus when linear time invariant system is subjected to a sinusoidal input it will produce sinusoidal output at steady state having same frequency as that of input. But the amplitude i.e. magnitude and phase angle of the output is different from that of input.

- Hence the effect of varying frequency on the magnitude and phase of the system is called frequency response of the system.

∴ The frequency domain transfer function of a system can be obtained by substituting 'jω' for 's' in the transfer function of the system.

- To get frequency response means to sketch the variation in magnitude and phase angle of $G(j\omega)$, when ω is varied from 0 to ∞.

Important Points to Remember

- $G(j\omega) = M \angle \phi$

  where M = Magnitude → $f(\omega)$, ϕ = Phase Angle → $f(\omega)$, ω = Input frequency

  Frequency response means to sketch variation in M and ϕ against ω. The stability of system can be decided from such frequency responses.

4.2 : Derivations of Resonant Peak and Resonant Frequency

Q.2 Derive the expressions for resonant peak M_r and resonant frequency ω_r for a standard second order system interms of ξ and ω_n.

☞ [SPPU : Dec.-02, 03, 08, 18, May-05, Marks 6]

Ans. : • Consider a standard second order system with the closed loop transfer function in time domain as,

$$\text{T.F.} \quad \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

- To get frequency domain T.F. replace 's' by 'jω' .

$$\frac{C(j\omega)}{R(j\omega)} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n j\omega + \omega_n^2} = \frac{\omega_n^2}{-\omega^2 + 2\xi\omega_n j\omega + \omega_n^2}$$

- Dividing by ω_n^2, $\dfrac{C(j\omega)}{R(j\omega)} = \dfrac{1}{\left[1-\left(\dfrac{\omega}{\omega_n}\right)^2\right] + 2\xi j \dfrac{\omega}{\omega_n}}$

- Replacing $\dfrac{\omega}{\omega_n} = x$, $\dfrac{C(j\omega)}{R(j\omega)} = \dfrac{1}{[1-x^2] + 2\xi j x}$

- In frequency response, the second order system shows a peak. This is called resonant peak M_r and corresponding frequency is called resonant frequency ω_r.

$$\text{Magnitude } \left|\frac{C(j\omega)}{R(j\omega)}\right| = \frac{1}{\sqrt{(1-x^2)^2 + 4\xi^2 x^2}}$$

- Find the value of x which maximises the magnitude i.e. $\frac{dM}{dx}=0$.

$$\frac{dM}{dx} = \frac{d}{dx}\left[\left((1-x^2)^2+4\xi^2x^2\right)^{\frac{-1}{2}}\right]$$

$$= -\frac{1}{2}\left[(1-x^2)^2+4\xi^2x^2\right]^{\frac{-3}{2}}\frac{d}{dx}\left[(1-x^2)^2+4\xi^2x^2\right]=0$$

- Solving, $4x[x^2+2\xi^2-1]=0$ i.e. $x=0$ or $x^2+2\xi^2-1=0$

$x^2=1-2\xi^2$ i.e. $x=\sqrt{1-2\xi^2}$ as $x = 0$ has no pratical significance

$\therefore \quad \frac{\omega}{\omega_n}=\sqrt{1-2\xi^2}$ which maximises M.

$$\therefore \quad \boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$ ... Resonant frequency

- The resonant peak is obtained by subtituting ω_r in expression of M_r,

$$M_r = \frac{1}{\sqrt{\left[1-\left(\sqrt{1-2\xi^2}\right)^2\right]^2+4\xi^2\left(\sqrt{1-2\xi^2}\right)^2}}$$

$$\therefore \quad \boxed{M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}}$$ ... Resonant peak

Q.3 If $G(s)H(s) = \frac{1}{s(s+1)}$, determine the value of : 1) Resonance peak and 2) Resonance frequency. **☞[SPPU : May-14, May-15, Marks 4]**

Ans. : The characteristics equation is $1 + G(s)H(s) = 0$

$\therefore \quad 1+\frac{1}{s(s+1)}=0$ i.e. $s^2+s+1=0$ i.e. $s^2+2\xi\omega_n s+\omega_n^2=0$

$\therefore \quad \omega_n^2=1,\ \omega_n=1$ and $2\xi\omega_n=1$, i.e. $\xi=0.5$.

1) $M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \mathbf{1.1547}$ ... Resonance peak

2) $\omega_r = \omega_n\sqrt{1-2\xi^2} = \mathbf{0.7071\ rad/sec.}$ ... Resonance frequency

4.3 : Co-relation Between Time Domain and Frequency Domain

Q.4 Explain the co-relation between time and frequency domain.
☞ [SPPU : Dec.-98,02,03, May-02,04,07,08,11, Marks 7,Dec.-15, 18, May-15,18, Marks 4

Ans. : The two important parameters associated with a standard second order system in frequency domain are,

i) **Resonant frequency (ω_r) :** This is the fequency at which the system shows a peak in its frequency response. It is given by, $\omega_r = \omega_n \sqrt{1-2\xi^2}$.

ii) **Resonant peak (M_r) :** It is the peak value of the frequency response of a second order system ocurring at resonant frequency ω_r and given by, $M_r = \dfrac{1}{2\xi\sqrt{1-\xi^2}}$.

- While in time domain it is known that for a standard second order system, damped frequency is $\omega_d = \omega_n \sqrt{1-\xi^2}$ and $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$.
- From these expressions co-relation between frequency domain and time domain can be obtained as,

1) Both M_p and M_r are the functions of the damping ratio alone. So both indicates the relative stability of the system.

2) As `ξ' increases , M_p decreases and gets vanished when $\xi = 1$. After that, system does not produce any overshoot.

   While in frequency domain M_r will vanish if, $\sqrt{1-2\xi^2} = 0$

   i.e. $2\xi^2 = 1$ i.e. $\xi^2 = 1/2$ $\therefore \xi = 0.707$

   And in such case system will not exhibit resonant peak. This can be shown in Fig. Q.4.1.

3) When ξ is very small i.e. less than 0.4 , both M_p and M_r will be very large and are not desirable. So `ξ ' is generally designed to be between $0.4 < \xi < 0.707$ where M_p and M_r are comparable to each other.

4) As $\xi \rightarrow 0$, M_p achieves 1 i.e. 100 % overshoot while M_r tries to approach to ∞ and hence both are undesirable from system point of view.

5) Also when ξ is between 0.4 to 0.707 both ω_r and ω_d are comparable to each other while when $\xi \rightarrow 0$, both ω_r and ω_d approaches the value ω_n. When 'ξ' is small, ω_d is large and hence rise time is small. Similarly the 'ξ' is small ω_r is large and hence system is more fast in the response. Hence ω_r indicates speed of the response.

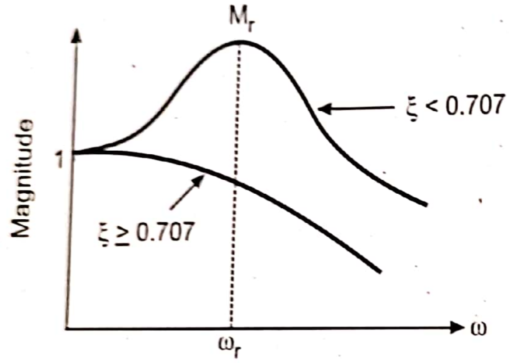


Fig. Q.4.1

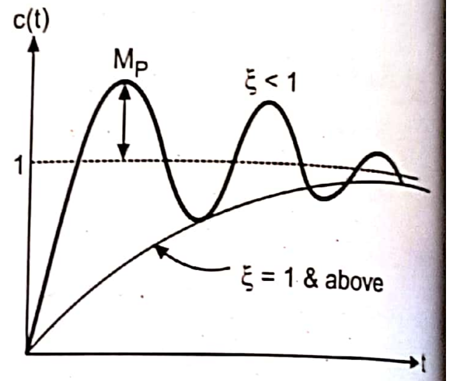


Fig. Q.4.2

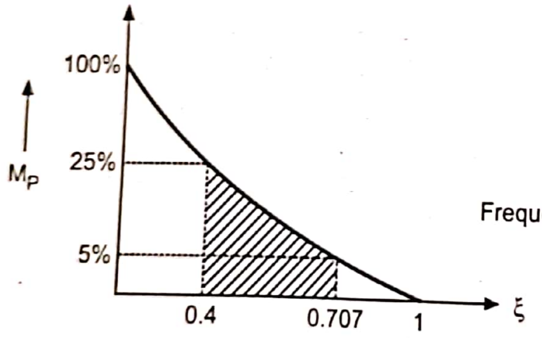


Fig. Q.4.3

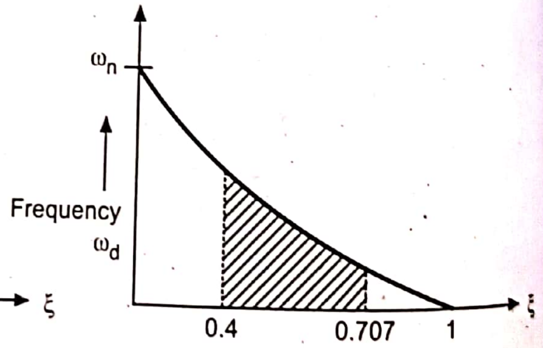


Fig. Q.24.4

- All these co-relations are shown in the Fig. Q.4.1, Q.4.2, Q.4.3 and Q.4.4.

- The two responses are comparable between $0.4 < \xi < 0.707$.

Q.5 Define bandwidth. State its expression for second order system.

Ans. : Bandwidth (B.W.) is the range of frequency upto ω_b which is called cut-off frequency at which the magnitude of the closed loop T.F. is 3 dB down from its zero frequency level.

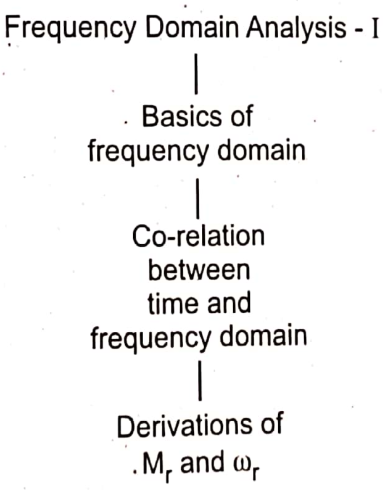
For standard second order system,

$$\boxed{\text{B. W.} = \omega_n \sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}}$$

Q.6 Define cutoff frequency. ☞ **[SPPU : Dec.-18, Marks 2]**

Ans. : The frequency at which the magnitude of the closed loop T.F. 3 dB down from its zero frequency level is caused cut-off frequency of that system.

Memory Map


Frequency Domain Analysis - I
Basics of frequency domain
Co-relation between time and frequency domain
Derivations of M_r and ω_r

END...

UNIT - 4

5

Frequency Domain Analysis - II

5.1 : Introduction to Bode Plot

Q.1 What is Bode plot ?

Ans. : • Bode plot consists of two plots which are,

1) Magnitude expressed in logarithmic values against logarithmic values of frequency called Magnitude plot.
2) Phase angle in degrees against logarithmic values of frequency called Phase angle plot.

The magnitude M $= |G(j\omega)H(j\omega)| = 20 \, \text{Log}_{10} \, |G(j\omega)| \, \text{dB}$

The phase angle $\phi = \angle G(j\omega)H(j\omega)$ in degrees

- The two plots are shown in the Fig. Q.1.1.

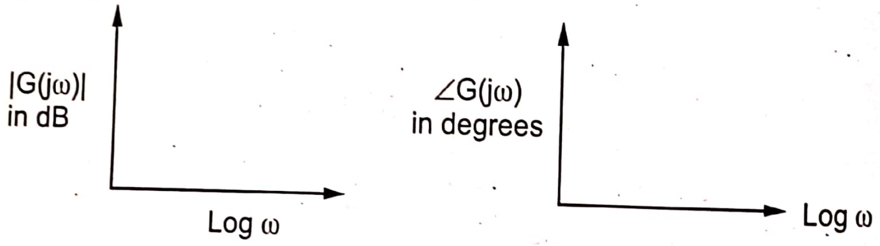


Fig. Q.1.1 Magnitude plot and Phase plot

- Both these plots are sktched on the same semi-log paper and together called Bode plot of the system.

Important Points to Remember

- A semi-log paper is used to sketch the Bode plot. In such paper the X-axis is divided into a logarithmic scale which is non linear one. While Y-axis is divided into linear scale and hence it is called ***semilog paper.***

- The logarithmic scale available on X-axis takes care of logarithmic value of ω There is no need to find the logarithmic values of frequency ω.

5.2 : Standard Form and Factors of G(jω)H(jω)

Q.2 What is the standard form of G(jω)H(jω) ? State the general factors of G(jω)H(jω) which contribute to the Bode plot.

Ans. : The standarad form of G(jω)H(jω) is called time constant form and given by,

$$\boxed{G(s)H(s) = \frac{K(1+T_1 s)(1+T_2 s)\ldots\ldots}{s^p\ (1+T_a s)(1+T_b s)}}$$

where K = Resultant system gain p = Type of the system

$T_1, T_2, T_a, T_b, \ldots\ldots$ = Time constants of different poles and zeros.

- Frequency domain transfer function can be obtained by substituting s = jω in the above expression,

$$\boxed{G(j\omega)H(j\omega) = \frac{K(1+T_1\, j\omega)\ (1+T_2\, j\omega)\ldots\ldots}{(j\omega)^p\ (1+T_a\, j\omega)(1+T_b\, j\omega)\ldots\ldots}}$$

- List of basic factors contributing Bode plot is ,

1) Resultant system gain K, constant factor. (When G(jω)H(jω) is expressed in time constant form).

2) Poles or zeros at the origin. (Integral and Derivative factors) i.e. $(j\omega)^{\pm p}$. Either poles or zeros at origin will be present.

3) Simple poles and zeros also called first order factors of the form $(1+sT)^{\pm 1}$ i.e. $(1+j\omega T)^{\pm 1}$

4) Quadratic factors i.e. quadratic pole or zero, which cannot be factorised into real factors i.e. having complex conjugate roots, of the form $\left(1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right)^{\pm 1} \approx \left[1+2\xi\, j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right]^{\pm 1}$

Q.3 Discuss the nature of Bode plot of i) Constant K ii) Pole at origin iii) Simple pole or zero iv) Quadratic pole or zero.
☞ [SPPU : Dec.-17, May-18, Marks 10]

Ans. :

Factor 1 : System gain K

- As gain `K' is constant, $20 \operatorname{Log}_{10} K$ is always constant though `ω is varied from 0 to ∞. So its magnitude plot will be straight line parallel to X-axis. While the angle of positive value of K is zero so K does not contribute to the phase angle plot.

  The effect of `K' is to shift the magnitude plot of $|G(j\omega)H(j\omega)|$ by a distance of 20 Log K dB upwards if K > 1 and downwards if K < 1.

- This fact is useful to design `K' for the required specifications.

Factor 2 : Pole at the origin (1/s) :

- The magnitude plot of one pole at the origin is the straight line of slope – 20 dB/decade passing through intersection point of $\omega = 1$ and 0 dB lines as shown in the Fig. Q.3.1.

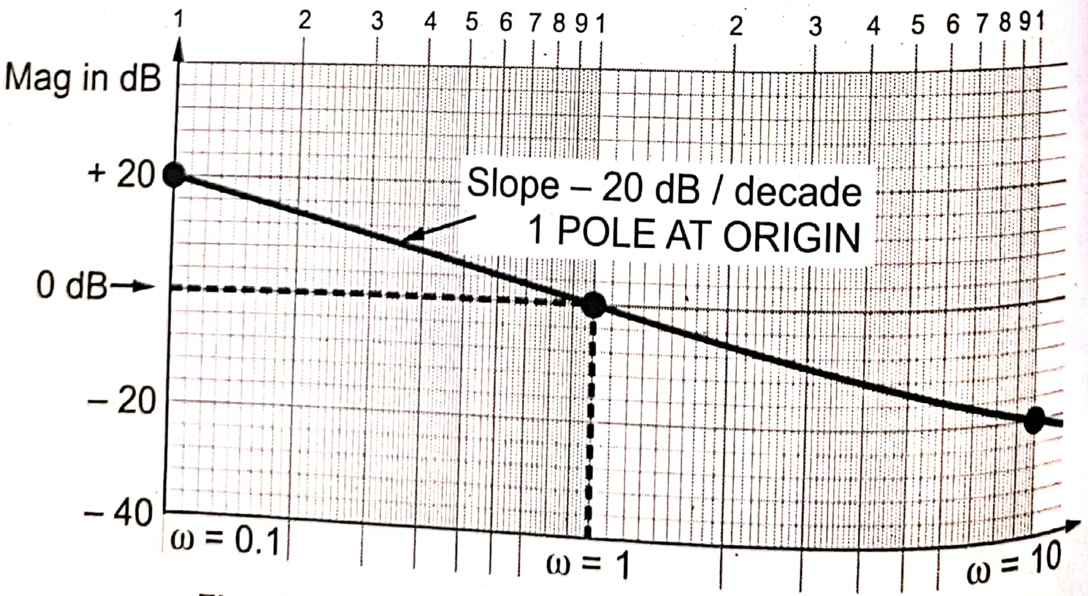


Fig. Q.3.1 Contribution by 1 pole at origin

If there are 2 poles at the origin, the slope of the straight line changes to – 40 dB/decade but passing through intersection point of $\omega = 1$ and 0 dB.

Therefore magnitude plot for 'P' poles at the origin gives a family of lines passing through intersection of ω = 1 and 0 dB line having slope [– 20 × P] dB/decade.

- **For zeros at origin the sign of the slope changes to positive.**
- In phase angle plot, each pole at the origin contributes fixed angle of – 90° at all the frequencies.

  In general P number of poles at the origin contribute – 90° × P angle to overall phase angle plot. This contribution is constant irrespective of 'ω'.
- **For zeros at origin the sign of the angle changes to positive.**

Factor 3 : **Simple Poles or Zeros** $(1+Ts)^{\pm 1}$ i.e. $(1+j\omega T)^{\pm 1}$

- The magnitude plot of simple pole is divided into two straight lines. One line of slope 0 dB/dec upto the critical frequency of the factor called **corner frequency** and second having slope – 20 dB/dec after the corner frequency.
- The frequency at which change of slope from 0 dB to – 20 dB/decade occurs is called **Corner frequency**, denoted by ω_C.

$$\boxed{\omega_C = 1/T \quad \text{for the simple pole or zero}}$$ ... **Corner frequency**

- **For a simple zero, the sign of the slope changes to positive, the nature remains same.**
- As the contribution of such factor is dominant after the corner frequecny, its effect is considered from its corner frequency onwards. Upto corner frequency such factor is as good as absent.
- For one simple pole the nature of the magnitude plot is shown in the Fig. Q.3.2.
- The magnitude plot obtained by this method is called **aymptotic Bode plot.**

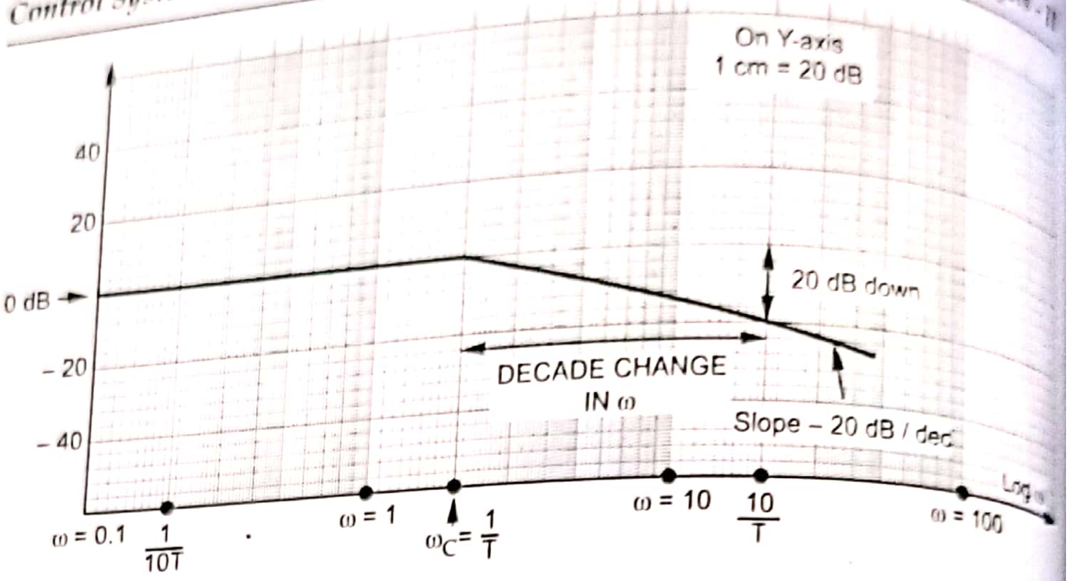


Fig. Q.3.2

The effect of such factors in overall magnitude plot is to change the slope of the resultant magnitude plot at every corner frequency by – 20 or +20 dB/dec depending on whether it is simple pole or zero.

- The phase angle of such factors is given by $\pm\tan^{-1}\omega T$ and its contribution is required to be calculated at various frequencies. The + sign for simple zero and – sign for simple pole.

Factor 4 : Quadratic Poles or Zeros

- These are represented as

$$\left(1+\frac{2\xi}{\omega_n}s+\frac{s^2}{\omega_n^2}\right)^{\pm 1} \quad \text{i.e.} \quad \left[1+2\xi\, j\left(\frac{\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2\right]^{\pm 1}$$

- For quadratic factor make sure that its roots are complex conjugates.
- The corner frequency of such factors is given by, $\omega_C = \omega_n$.
- The magnitude plot of a quadratic pole is divided into two straight lines. One line of slope 0 dB/dec upto the corner **frequency** ω_n and second having slope **– 40 dB/dec** after the corner frequency ω_n.

- So such a quadratic factor changes the slope of the resultant magnitude plot by ± 40 dB/dec. Use + sign for the quadratic zero and – sign for the quadratic pole.
- The phase angle of the quadratic pole is given by,

$$\phi = -\tan^{-1}\left\{\frac{2\xi\left(\frac{\omega}{\omega_n}\right)}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right\}$$

Important Points to Remember

- Note that at $\omega = \omega_C = \omega_n$, the phase angle is $-90°$ and due to second order its contribution must tend to $-180°$ as $\omega \to \infty$. For $\omega > \omega_n$, the angle comes out to be positive by above formula. In such a case the angle contribution obtained must be considered, by subtracting 180 ° from the positive ϕ obtained by above formula.
- **When the magnitude plot of two factors representing straight lines are to be added together in the Bode plot, then resultant line always has a slope which is algebraic addition of the individual slopes of the two lines which are to be added.**
- **The starting slope of the Bode plot for the function G(s)H(s) gets decided by number of poles or zeros at origin present in G(s)H(s).**

5.3 : Frequency Domain Specifications

Q.4 Discuss the various frequency domain specifications.

☞ [JNTU : Dec.-01, 06, 10, 11, May-01, 04, 09, 12, Marks 6]

OR Define the following : i) Resonant peak ii) Resonant frequency iii) Gain cross-over frequency iv) Phase cross-over frequency v) Gain Margin (G.M.) vi) Phase margin (P.M.).

☞ [JNTU : May-01, 05, 16, 17, 18, Dec.-12, 16, Marks 6]

Ans. : i) **Resonant peak (M_r) :** It is the maximum value of magnitude of the closed loop frequency response. Larger the value of resonant peak more is the value of peak overshoot of system for step input.

ii) **Resonant frequency (ω_r) :** The frequency at which resonant peak M_r occurs in closed loop frequency response is called resonant frequency.

iii) **Gain crossover frequency (ω_{gc}) :** The frequency at which magnitude of $G(j\omega)H(j\omega)$ is unity i.e. 1 is called **gain crossover frequency.**

- Generally magnitude of $G(j\omega)H(j\omega)$ is expressed in dB. And dB value of 1 is $20 \text{Log}_{10} 1 = 0$ dB.
- Thus gain cross-over frequency is the frequency at which magnitude plot crosses the 0 dB line. Hence at ω_{gc}, magnitude of $G(j\omega)H(j\omega)$ is 0 dB.

iv) **Phase crossover frequency (ω_{pc}) :** The frequency at which phase angle of $G(j\omega)H(j\omega)$ is $-180°$ is called **phase crossover frequency,** ω_{pc}.

v) **Gain margin G.M. :** As gain 'K' is increased, the system stability reduces and for a certain value of 'K' it becomes marginally stable.

- Gain margin is defined as the margin in gain K allowable by which gain can be increased till system reaches on the verge of instability.
- Mathematically it can be defined as reciprocal of the magnitude of the $G(j\omega)H(j\omega)$ measured at phase crossover frequency.

$$\therefore \quad \text{G.M.} = \frac{1}{|G(j\omega)H(j\omega)|_{\omega = \omega_{pc}}}$$

In decibels

$$\boxed{\begin{array}{l} \text{G.M.} = 20 \text{ Log} \dfrac{1}{|G(j\omega)H(j\omega)|_{\omega = \omega_{pc}}} \\[2ex] \text{G.M.} = -20 \text{ Log}_{10} |G(j\omega)H(j\omega)|_{\omega = \omega_{pc}} \end{array}}$$

- More positive the G.M., more stable is the system.

ii) **Phase margin P.M.** : Similar to the gain, it is possible to introduce phase lag in the system i.e. negative angles without affecting magnitude plot of $G(j\omega)H(j\omega)$.

- The amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability is called **phase margin P.M.**
- Mathematically it can be defined as,

$$P.M. = [\angle G(j\omega)H(j\omega)|_{\text{at } \omega = \omega_{gc}}] - [(-180^\circ)]$$

$$\therefore \quad \boxed{P.M. = 180^\circ + \angle G(j\omega)H(j\omega)|_{\omega = \omega_{gc}}}$$

- Positive P.M. indicates stable system while negative P.M. indicates unstable system. More positive the P.M., more stable is the system.

5.4 : G.M. and P.M. from Bode Plot and Stability Conditions

Q.5 Explain the procedure to obtain G.M. and P.M. from the Bode plot. Draw the Bode plot for stable and unstable conditions.

☞ [JNTU : Dec.-10, 11, 12, May-16, 17, 18, Marks 6]

Ans. : • In a Bode plot, identify $\omega = \omega_{pc}$ and Extend $\omega = \omega_{pc}$ line upwards till it intersects resultant magnitude plot at point A. The magnitude corresponding to point A is $|G(j\omega)H(j\omega)|_{\omega = \omega_{pc}}$

- The difference between 0 dB and magnitude corresponding to point A i.e. $|G(j\omega)H(j\omega)|_{\omega = \omega_{pc}}$ is **Gain Margin.** If point A is below 0 dB, G.M. is positive and if point A is above 0 dB, G.M. is negative.
- For phase margin, identify $\omega = \omega_{gc}$ and extend $\omega = \omega_{gc}$ line downwards till it intersects phase angle plot at point C. The angle corresponding to the point C is $\angle|G(j\omega)H(j\omega)|_{\omega = \omega_{gc}}$.

The distance between this point C i.e. $\angle|G(j\omega)H(j\omega)|_{\omega = \omega_{gc}}$ and - 180° line is nothing but the phase margin. If point C is above

−180° line, P.M. is positive and if it is below −180° line, P.M. is negative.

- System is said to be stable when P.M. and G.M. are positive while system is said to be unstable when both P.M. and G.M. are negative. Now when system is on the verge of instability i.e. marginally stable in nature then G.M. and P.M. both are zero. This is possible when $\omega_{gc} = \omega_{pc}$. b
- All these conditions are shown in the Fig. Q.5.1, Q.5.2 and Q.5.3.

Stability Conditions :

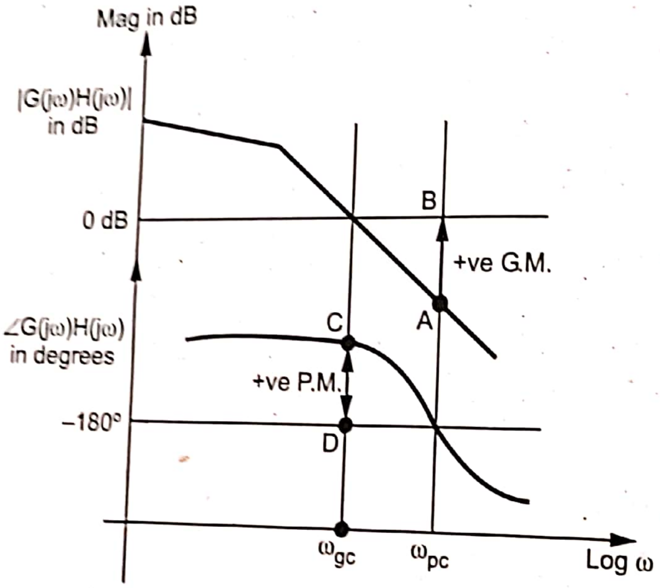

Mag in dB
|G(jω)H(jω)| in dB
0 dB
B
+ve G.M.
A
C
∠G(jω)H(jω) in degrees
+ve P.M.
−180°
D
ω_gc
ω_pc
Log ω

Fig. Q.5.1 $\omega_{gc} < \omega_{pc}$ G.M. and P.M. positive, stable system

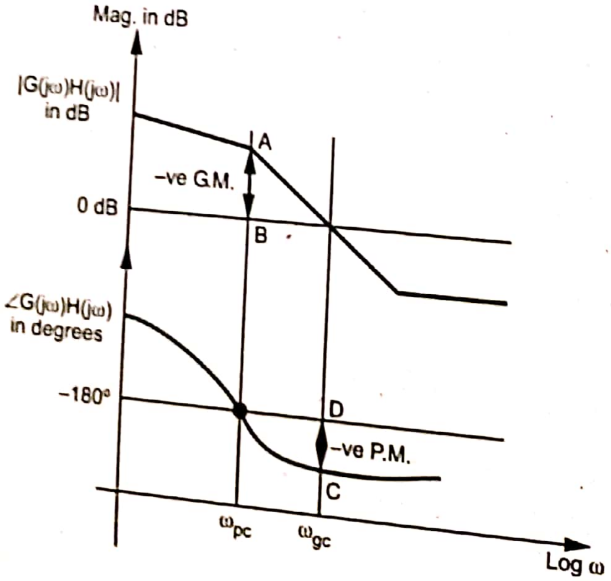

Mag. in dB
|G(jω)H(jω)| in dB
A
−ve G.M.
0 dB
B
∠G(jω)H(jω) in degrees
−180°
D
−ve P.M.
C
ω_pc
ω_gc
Log ω

Fig. Q.5.2 $\omega_{gc} > \omega_{pc}$ G.M. and P.M. negati

$-180°$ line, P.M. is positive and if it is below $-180°$ line, P.M. is negative.

- System is said to be stable when P.M. and G.M. are positive while system is said to be unstable when both P.M. and G.M. are negative. Now when system is on the verge of instability i.e. marginally stable in nature then G.M. and P.M. both are zero. This is possible when $\omega_{gc} = \omega_{pc}$. b
- All these conditions are shown in the Fig. Q.5.1, Q.5.2 and Q.5.3.

Stability Conditions :

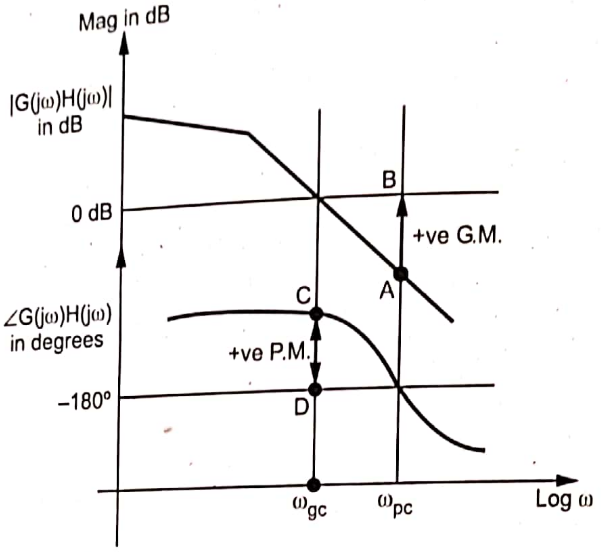

Mag in dB
|G(jω)H(jω)| in dB
0 dB
B
+ve G.M.
A
C
∠G(jω)H(jω) in degrees
+ve P.M.
–180°
D
ωgc
ωpc
Log ω

Fig. Q.5.1 $\omega_{gc} < \omega_{pc}$ **G.M. and P.M. positive, stable system**

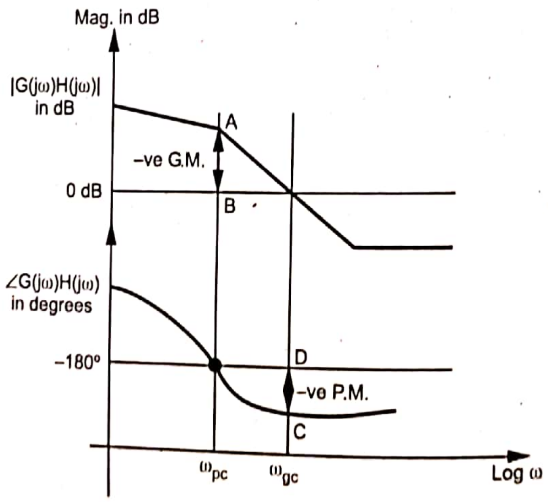

Mag. in dB
|G(jω)H(jω)| in dB
A
–ve G.M.
0 dB
B
∠G(jω)H(jω) in degrees
–180°
D
–ve P.M.
C
ωpc
ωgc
Log ω

Fig. Q.5.2 $\omega_{gc} > \omega_{pc}$ **G.M. and P.M. negative, unstable system**

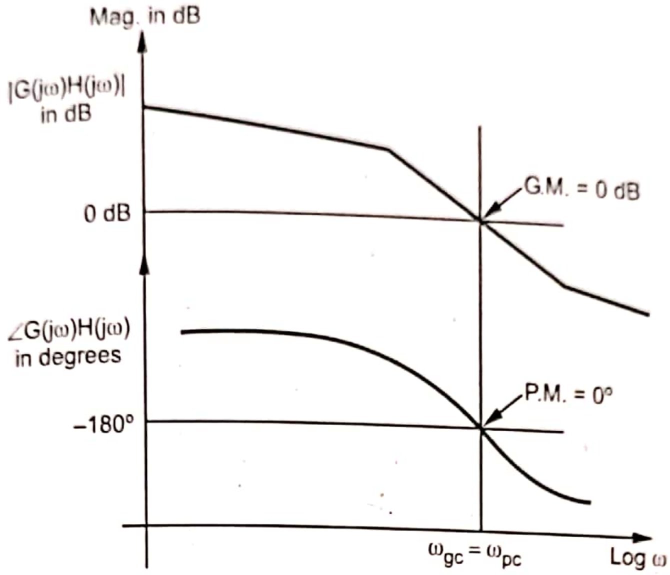


Fig. Q.5.3 $\omega_{gc} = \omega_{pc}$ **G.M. and P.M. zero, marginally stable system**

5.5 : Steps to Sketch Bode Plot

Important Points to Remember

1) Express given G(s)H(s) into time constant form.
2) Draw a line of 20 Log K dB which is parallel to Log ω axis i.e. X-axis.
3) Draw a line of appropriate slope representing poles or zeros at the origin, passing through intersection point of ω = 1 and 0 dB. For 1 pole at the origin − 20 dB/dec, for 2 poles at the origin − 40 dB/dec and so on.
4) Shift the intersection point of ω = 1 and 0 dB on 20 Log K line and draw parallel line to the line drawn in step 3. This is addition of constant K and number of poles or zeros at the origin.
5) Change the slope of this line at various corner frequencies by appropriate value i.e. depending upon which factor is occurring at corner frequency. For a simple pole, slope must be changed by − 20 dB/decade, for a simple zero by + 20 dB/decade etc. **Do not draw these individual lines.** Change the slope of line obtained in step 5 by respective value and draw line with resultant slope. Continue this line till it intersects next corner frequency line. Change the slope and continue. Apply necessary correction for quadratic factor if required.

6) Prepare the phase angle table and obtain the table of ω and resultant phase angle ϕ_R by actual calculation. Plot these points and draw the smooth curve obtaining the necessary phase angle plot.

- **Remember that at every corner frequency slope of resultant line must change.**

7) Determine ω_{gc}, ω_{pc}, G.M. and P.M. from the Bode plot and predict the stability of the system.

Q.6 Sketch the Bode plot of the transfer function and determine PM and GM from the plot,

$$G(s)H(s) = \frac{30}{s(1+0.5s)(1+0.08s)}$$

☞ **[SPPU : Dec.-16, Marks 10]**

Ans. : Step - 1 : G(s)H(s) in time constant form

Step - 2 : Factors

i) K = 30, 20 log K = 29.54 dB, straight line parallel to log ω axis.

ii) $\frac{1}{s}$, one pole at origin, straight line of slope −20 dB/dec

iii) $\frac{1}{1+0.5s}$, simple pole, $T_1 = 0.5$, $\omega_{C1} = \frac{1}{T_1} = 2$, straight line of slope −20 dB/dec for $\omega \geq 2$

iv) $\frac{1}{1+0.08s}$, simple pole, $T_2 = 0.08$, $\omega_{C2} = \frac{1}{T_2} = 12.5$, straight line of slope −20 dB/dec for $\omega \geq 12.5$

Range of ω	$0 < \omega < 2$	$2 \leq \omega < 12.5$	$12.5 \leq \omega < \infty$
Resultant slope in dB/dec	−20	−20 −20 = −40	−40 −20 = −60

Step - 3 : Phase angle table,

$$G(j\omega)\,H(j\omega) = \frac{30}{j\omega\,(1+0.5\,j\omega)\,(1+0.08\,j\omega)}$$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}0.5\omega$	$-\tan^{-1}0.08\omega$	ϕ_R
2	$-90°$	$-45°$	$-9.09°$	$-144.1°$
8	$-90°$	$-75.9°$	$-32.6°$	$-198.5°$
12.5	$-90°$	$-80.9°$	$-45°$	$-215.9°$
∞	$-90°$	$-90°$	$-90°$	$-270°$

Step - 4 : Bode plot is shown in the Fig. Q.6.1. From it, (See Fig. Q.6.1 on next page).

G.M. = -8 dB and P.M. = $-15°$

The system is **unstable**

Q.7 Find the stability of the following system by sketching the Bode plot, $G(s) = \dfrac{200}{s(s+10)(s+2)}$ ☞ **[SPPU : Dec.-17, Marks 12]**

Ans. : Step - 1 : G(s) in time constant form

$$G(s) = \frac{10}{s(1+0.5s)(1+0.1s)}$$

Step - 2 : Factors

i) K = 10, 20 log K = 20 dB, straight line parallel to log ω axis.

ii) $\frac{1}{s}$, one pole at origin, straight line of slope -20 dB/dec

iii) $\frac{1}{1+0.5s}$, simple pole, $\omega_{C1} = \frac{1}{0.5} = 2$, straight line of slope -20 dB/dec for $\omega \geq 2$

iv) $\frac{1}{1+0.1s}$, simple pole, $\omega_{C2} = \frac{1}{0.1} = 10$, straight line of slope -20 dB/dec for $\omega \geq 10$

Range of ω	$0 < \omega < 2$	$2 \leq \omega < 10$	$10 \leq \omega < \infty$
Resultant slope in dB/dec	-20	$-20-20=-40$	$-40-20=-60$

Step - 3 : Phase angle table,

$$G(j\omega) = \frac{10}{j\omega(1+0.5j\omega)(1+0.1j\omega)}$$

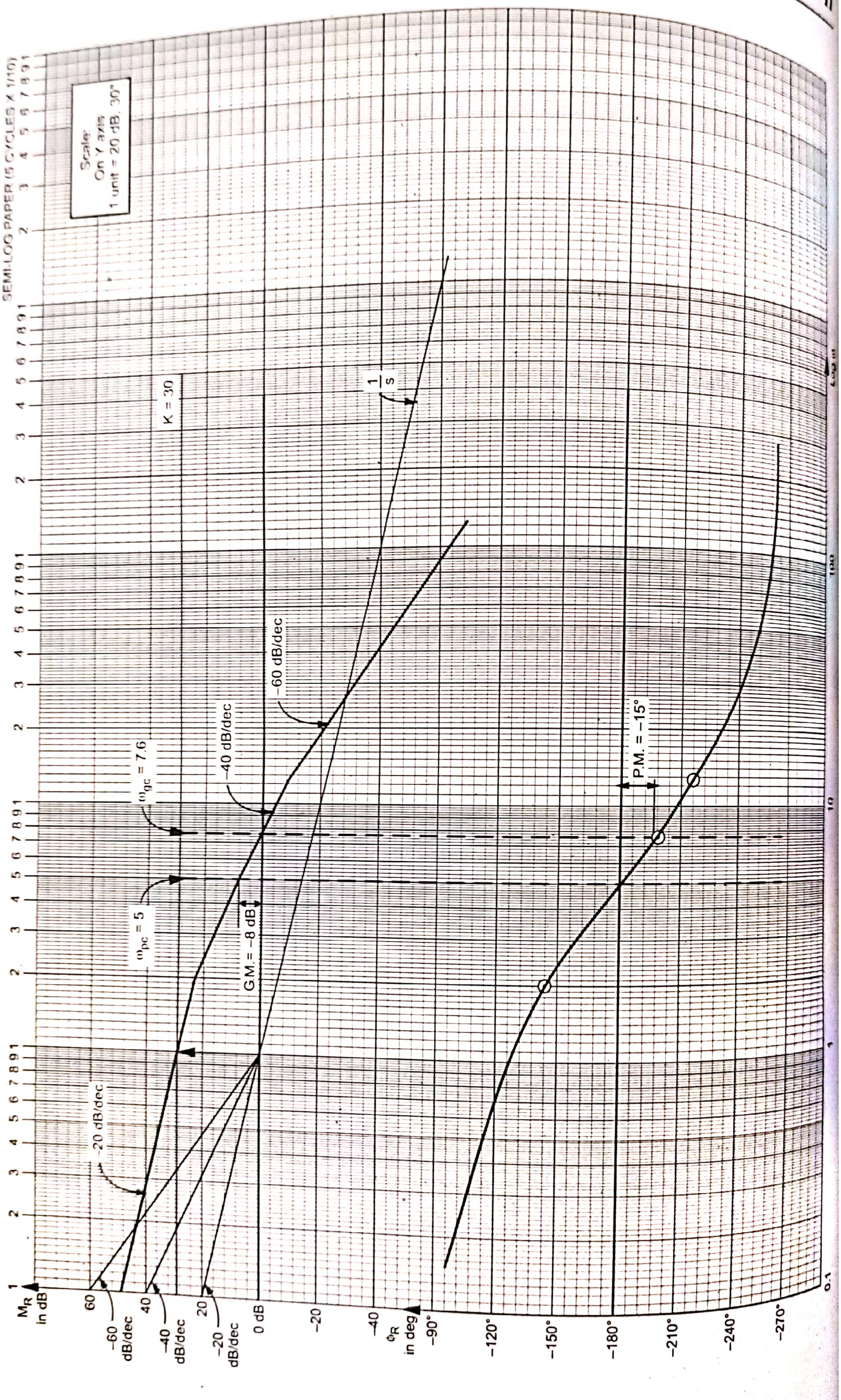

SEMI-LOG PAPER (5 CYCLES X 1/10)
Scale:
On Y axis
1 unit = 20 dB, 30°
K = 30
$\frac{1}{s}$
–60 dB/dec
–40 dB/dec
–20 dB/dec
ω_{gc} = 7.6
ω_{pc} = 5
G.M. = –8 dB
P.M. = –15°
M_R in dB
60
40
20
0 dB
–20
–40
–60 dB/dec
–40 dB/dec
–20 dB/dec
ϕ_R in deg
–90°
–120°
–150°
–180°
–210°
–240°
–270°
0.1
1
10
100

Fig. Q.6.1

ω	$\frac{1}{j\omega}$	$-\tan^{-1}0.5\omega$	$-\tan^{-1}0.1\omega$	ϕ_R
0.2	−90 °	−5.71°	−1.14°	− 96.8°
2	−90 °	−45°	−11.3°	−146.3°
8	−90 °	−75.96°	−38.6°	−204.6°
∞	−90 °	−90°	−90°	−270°

Step - 4 : The Bode plot is shown in the Fig. Q.7.1 (See Fig. Q.7.1 on next page). The system is **stable** as G.M. and P.M. are positive.

Q.8 Draw Bode plot for following system $G(s) = \frac{40}{s(s+3)(s+12)}$. Find gain margin and phase margin comment on stability.

☞ **[SPPU : May-15, Marks 12]**

Ans. : Refer Q.7 for procedure and verify the answers,

$\omega_{C1} = 3$, $\omega_{C2} = 12$, $K = 1.111$, $20 \log K = 0.915$ dB,

G.M. = **+ 22 dB,** P.M. = **+ 58°**, system **stable.**

Q.9 The open loop unity feedback system is,

$G(s) = \frac{2000}{s(s+1)(s+100)}$ Draw Bode plot, determine the followig and comment on the stability.

i) Gain cross over frequency

ii) Phase cross over frequency

iii) Gain margin

iv) Phase margin.

☞ **[SPPU : May-18, Marks 12]**

Ans. : Refer Q.7 for the procedure and verify the answers as,

$K = 20$, $\omega_{C1} = 1$, $\omega_{C2} = 100$,

i. ω_{gc} = **4 rad/sec,**

ii. ω_{pc} = **10 rad/sec**

iii.G.M. = **+ 14 dB,**

iv.P.M. = **+ 12 °**

The system is **stable.**

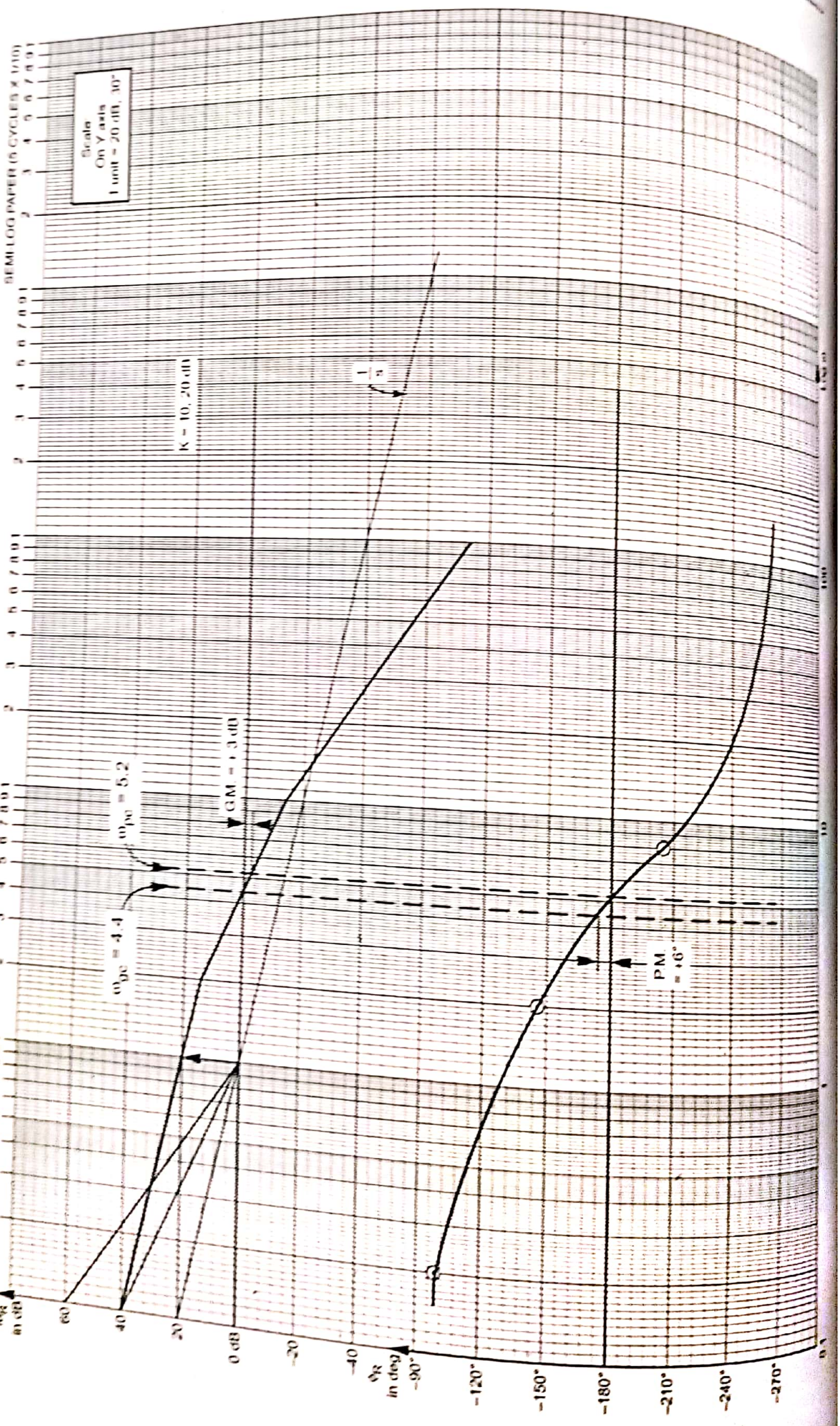

SEMILOG PAPER (5 CYCLES × 1/10)
Scale
On Y axis
1 unit = 20 dB, 30°
K = 10, 20 dB
$-\frac{1}{s}$
GM = +3 dB
ω_{pc} = 5.2
ω_{gc} = 4.4
PM = +6°
M_{dB} in dB
60
40
20
0 dB
-20
-40
ϕ_R in deg
-90°
-120°
-150°
-180°
-210°
-240°
-270°
0.1
1
10
100
1000

Fig. Q.7.1

Q.10 Draw Bode plot for following system, $G(S) = \frac{1000}{s(1+0.1s)(1+0.001s)}$. Find gain margin and phase margin comment on stability.

☞ **[SPPU : May-16, Marks 12]**

Ans. : Refer Q.7 for the procedure and verify the answer as, $K = 1000$, $\omega_{C1} = 10$, $\omega_{C2} = 1000$. From the Bode plot, $\omega_{gc} = \omega_{pc} = 100$ rad/sec, hence G.M. = 0 dB, P.M. = 0 °, and the system is **marginally stable** in nature.

Q.11 Draw Bode plot for following system $G(s) = \frac{20(s+2)}{s(s+10)}$ Find gain margin and phase margin comment on stability.

☞ **[SPPU : May-17, Marks 12]**

Ans. : Step - 1 : G(s) in time constant form.

$$G(s) = \frac{4(1+0.5s)}{s(1+0.1s)}$$

Step - 2 : Factors

i) $K = 4$, $20 \log K = 12.04$ dB, straight line parallel to $\log \omega$ axis.

ii) $\frac{1}{s}$, one pole at origin, straight line of slope −20 dB/dec.

iii) $(1+0.5s)$, simple zero, $\omega_{C1} = \frac{1}{0.5} = 2$, straight line of slope +20 dB/dec for $\omega \geq 2$.

iv) $\frac{1}{1+0.1s}$, simple pole, $\omega_{C2} = \frac{1}{0.1} = 10$, straight line of slope −20 dB/dec for $\omega \geq 10$.

Range of ω	$0 < \omega < 2$	$2 \leq \omega < 10$	$10 \leq \omega < \infty$
Resultant slope in dB/dec	−20	−20 + 20 = 0	0 − 20 = −20

Step - 3 : $G(j\omega) = \frac{4(1+0.5j\omega)}{j\omega(1+0.1j\omega)}$, phase angle table is,

ω	$\frac{1}{j\omega}$	$-\tan^{-1} 0.5\omega$	$-\tan^{-1} 0.1\omega$	ϕ_R
0.2	−90°	+5.71°	−1.14°	−85.4°
2	−90°	+45°	−11.3°	−56.3°
10	−90°	+78.69°	−45°	−56.3°
100	−90°	+88.8°	−84.28°	−85.4°
∞	−90°	+90°	−90°	−90°

Step - 4 : The Bode plot is shown in the Fig. Q.11.1. (See Fig. Q.11.1 on next page). from the plot,

G.M. = +∞ dB, P.M. = + 116°.

Q.12 The open loop unity feedback system is

$$G(s) = \frac{K}{s(s+1)(s+10)}$$

Draw Bode plot and determine

I) The value of K for gain margin of 7 dB.

II) The value of K for phase margin of 40 °.

☞ **[SPPU : May-18, Marks 12]**

Ans. : Step - 1 : G(s)H(s) in time constant form.

$$G(s)H(s) = \frac{0.1K}{s\,(1+s)(1+0.1s)} = \frac{A}{s\,(1+s)(1+0.1s)}$$

Step - 2 : Factors

i) A is unknown, its effect is to shift magnitude plot upwards or downwards by 20 log A dB.

ii) $\frac{1}{s}$, one pole at origin, straight line of slope −20 dB/dec.

iii) $\frac{1}{1+s}$, simple pole, $\omega_{C1} = 1$, straight line of slope −20 dB/dec for $\omega \geq 1$.

iv) $\frac{1}{1+0.1s}$, simple pole, $\omega_{C2} = \frac{1}{0.1} = 10$, straight line of slope −20 dB/dec for $\omega \geq 10$.

Draw the Bode plot without A.

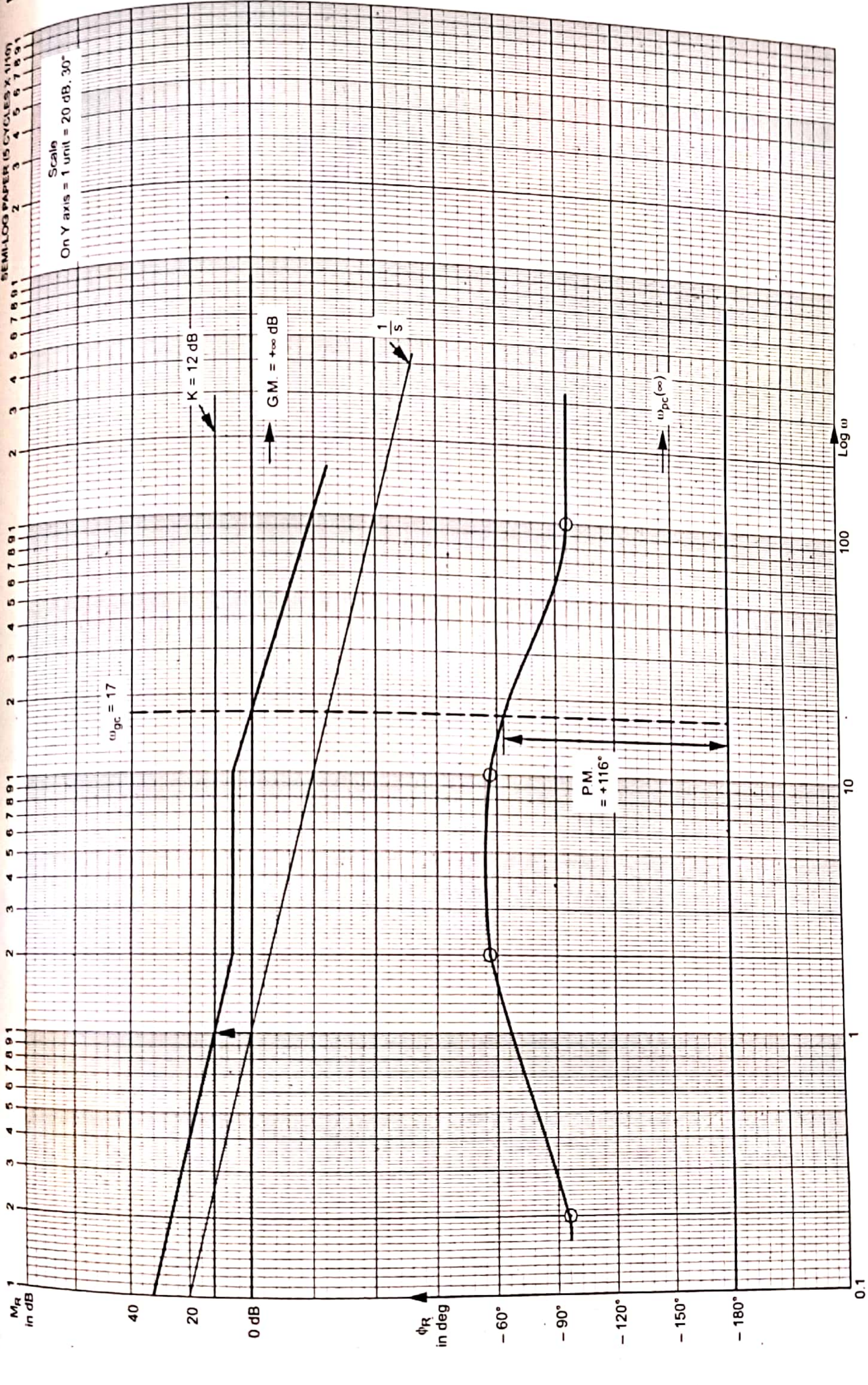


Fig. Q.11.1

Range of ω	$0 < \omega < 1$	$1 \leq \omega < 10$	$10 \leq \omega < \infty$
Resultant slope in dB/dec	-20	$-20 - 20 = -40$	$-40 - 20 = -60$

Step - 3 : Phase angle table,

$$G(j\omega)\,H(j\omega) = \frac{A}{j\omega\,(1+j\omega)\,(1+0.1j\omega)},$$

ω	$-90°$	$-\tan^{-1}\omega$	$-\tan^{-1}0.1\omega$	ϕ_R
0.1	$-90°$	$-5.71°$	$-0.57°$	$-96.28°$
1	$-90°$	$-45°$	$-5.71°$	$-140.7°$
5	$-90°$	$-78.7°$	$-26.56°$	$-195.26°$
∞	$-90°$	$-90°$	$-90°$	$-270°$

Draw the Bode plot without effect of A as shown in the Fig. Q.12.1

Steps to find K for given G.M. :

1) Draw the horizontal line below 0 dB at a distance of given G.M., till it intersects vertical line of ω_{pc}. This is point B.
2) Draw the vertical line from ω_{pc}, till it intersects magnitude plot without K. This is point A.
3) The point A must be at B to match given G.M. Hence A to B is the shift required i.e. contribution by K which is 20 Log K dB.
4) Upward shift must be taken positive and downward shift negative.

$$20 \text{ Log } K = \text{Shift (AB)}$$

i) From the plot, shift (AB) = + 13 dB ↑

This is effect of A.

$\therefore$ 20 log A i.e. A = 4.467 = 0.1 K

$\therefore$ K = **44.67** (for G.M. = + 7dB)

Steps to find K for given P.M. :

1) Draw the horizontal line above −180° line at a distance of given P.M., till it intersects phase angle plot. This is point F.
2) Draw the vertical line from F, till it intersects the magnitude plot without K. This is point C.

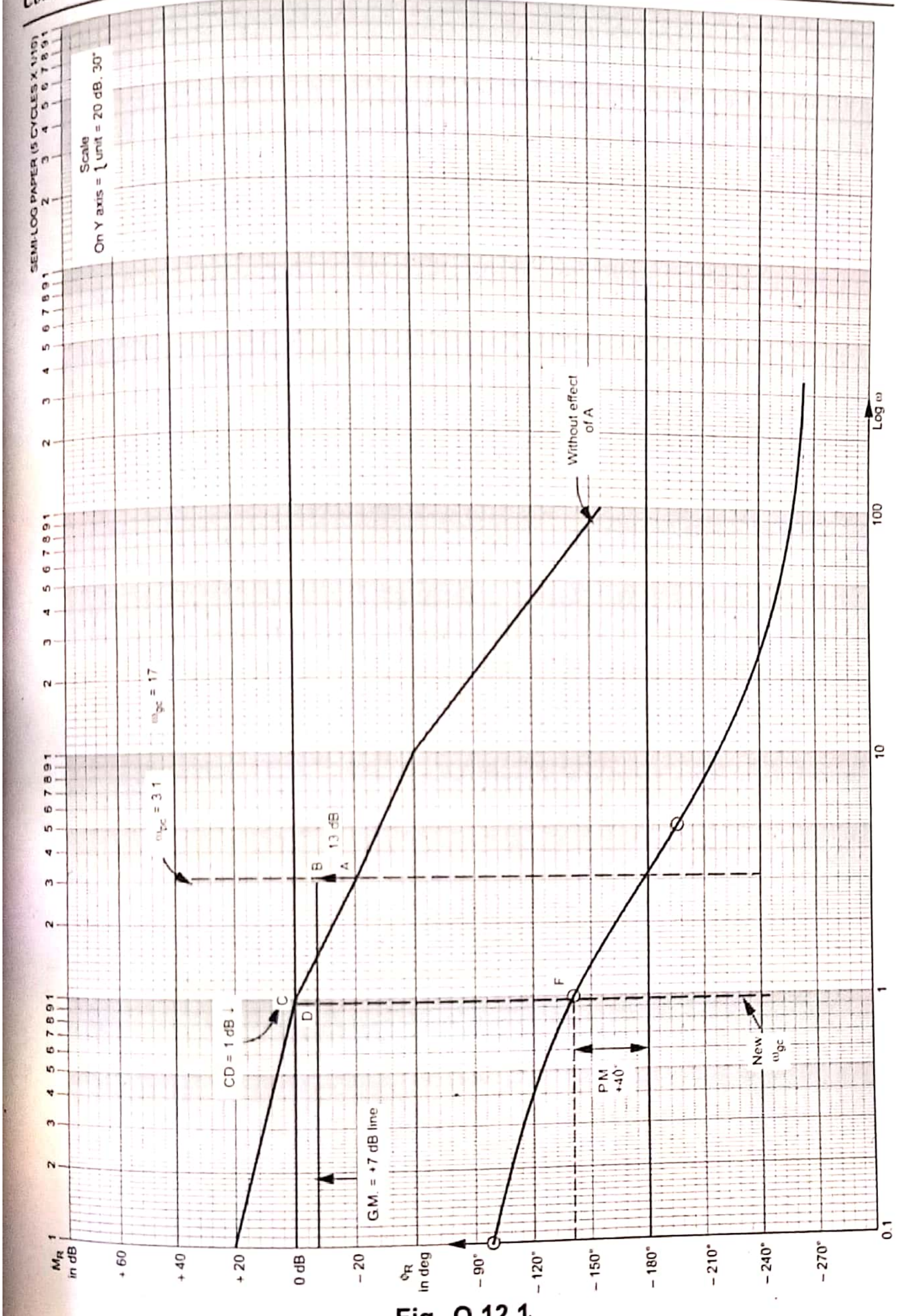


Fig. Q.12.1

3) This C must be on 0 dB line at D, as the vertical line through point F must be ω_{gc} line for given P.M.

4) Thus distance of C from 0 dB line is the shift required to satisfy given P.M. This must be 20 Log K dB.

5) Upward shift (CD) must be taken positive while downward shift (CD) must be taken negative.

$$20 \text{ Log } K = \text{Shift (CD)}$$

ii) Given P.M. = 40°, so from the plot for 40° of P.M., the magnitude plot is to be shifted through distance CD, which is the effect of A.

Shift (CD) = 1 dB↓ i.e. $20 \log A = -1$

$\therefore$ $A = 0.89 = 0.1K$

$\therefore$ $K = $ **8.91** for P.M. = 40°

5.6 : Advantages of Bode Plot

Q.13 Give the advantages of Bode plot. ☞ **[SPPU : Dec.-11, Marks 4]**

Ans. : 1) It shows both low and high frequency characteristics of transfer function in single diagram.

2) The plots can be easily constructed using some valid approximations.

3) Relative stability of system can be studied by calculating G.M. and P.M. from the Bode plot.

4) The various other frequency domain specifications like cut-off frequency, bandwidth etc. can be determined.

5) Data for constructing complicated polar and Nyquist plots can be easily obtained from Bode plot.

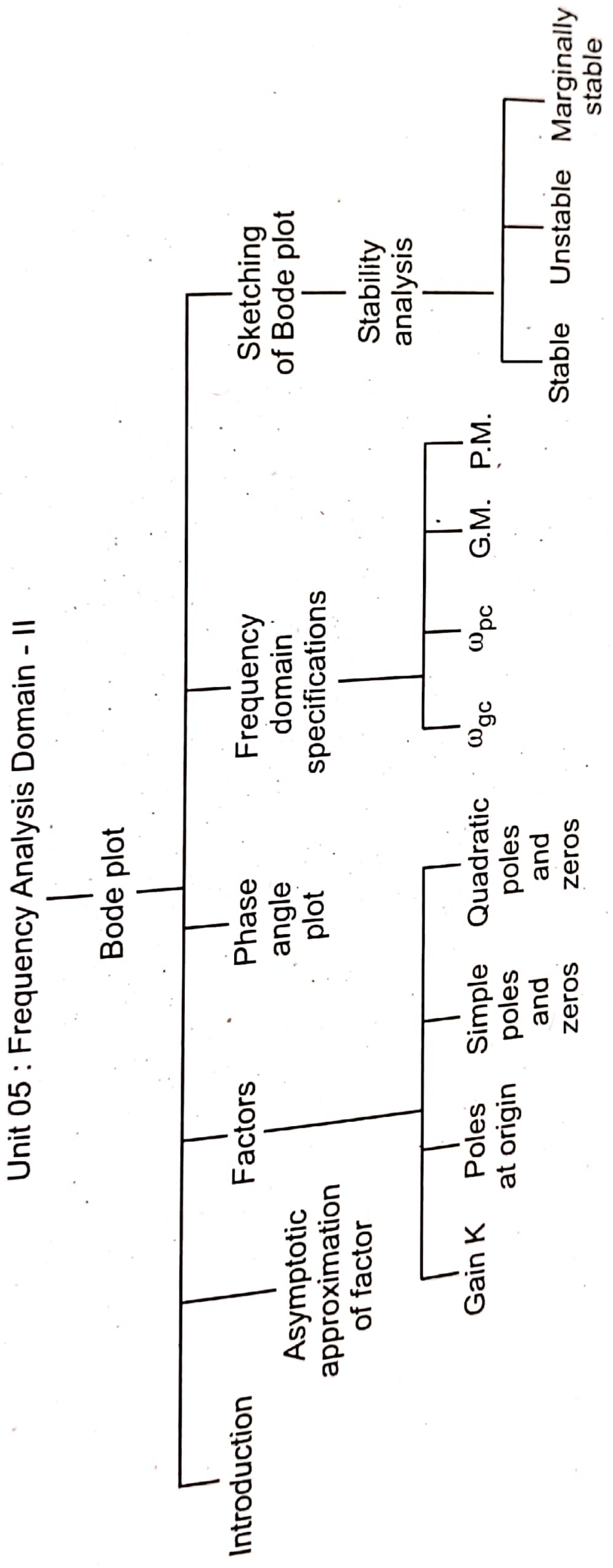
6) Transfer function of system can be obtained from Bode plot.

7) It indicates how system should be compensated to get the desired response.

8) The value of system gain K can be designed for required specifications of G.M. and P.M. from Bode plot.

9) Without the knowledge of the transfer function the Bode plot of stable open loop system can be obtained experimentally.

Memory Map


Unit 05 : Frequency Analysis Domain - II
Bode plot
Introduction
Asymptotic approximation of factor
Factors
Gain K
Poles at origin
Simple poles and zeros
Quadratic poles and zeros
Phase angle plot
Frequency domain specifications
ω_{gc}
ω_{pc}
G.M.
P.M.
Sketching of Bode plot
Stability analysis
Stable
Unstable
Marginally stable

END...

UNIT - 4

6 Polar and Nyquist Plot

6.1 : Polar Plot

Q.1 What is polar plot ? Explain polar plots for Type 0, 1 and 2 systems.
☞ [SPPU : Dec.-17, Marks 4]

Ans. :

- Polar plot is defined as the locus of tips of the phasors of various magnitudes plotted at the corresponding phase angles for different values of frequencies from 0 to ∞.
- The polar plot starts at point representing magnitude and phase angle for $\omega = 0$. While it terminates at a point representing magnitude and phase angle for $\omega = \infty$.

Type 0 system : Consider a open loop transfer function

$$G(s)H(s) = \frac{1}{1+Ts}.$$

$$G(j\omega)H(j\omega) = \frac{1}{1+Tj\omega} = \frac{1+j0}{1+j\omega T}$$

$$|G(j\omega)H(j\omega)| = M = \frac{1}{\sqrt{1+\omega^2 T^2}}, \quad \angle G(j\omega)H(j\omega) = \phi = -\tan^{-1}(\omega T)$$

ω	M	ϕ
0	1	0 °
∞	0	− 90 °

- The rotation of the plot is $-90° - 0° = -90°$ i.e. 90° clockwise. So the polar plot is starting from $1\angle 0°$ it ends at $0\angle -90°$ with 90° clockwise rotation as shown in the Fig. Q.1.1.

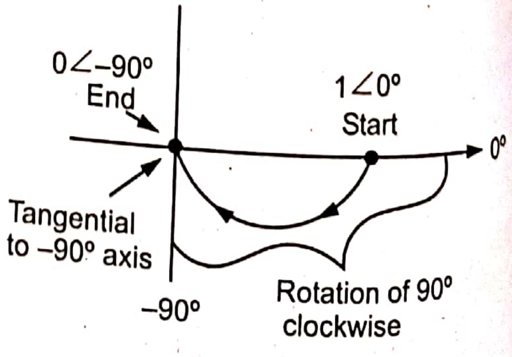


Fig. Q.1.1

Type 1 system : Consider a open loop transfer function

$$G(s)H(s) = \frac{1}{s(1+Ts)}$$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega(1+Tj\omega)} = \frac{1+j0}{(0+j\omega)(1+Tj\omega)}$$

$$\therefore \quad M = \frac{1}{\omega \times \sqrt{1+\omega^2 T^2}}, \quad \phi = -90° - \tan^{-1}\omega T$$

ω	M	ϕ
0	∞	− 90°
∞	0	− 180°

- The rotation of the plot is,

  $-180° - (-90°) = -90°$ i.e. 90° clockwise.

- So the polar plot is starting from $\infty\angle-90°$ it ends at $0\angle-180°$ with 90° clockwise rotation as shown in the Fig. Q.1.2.

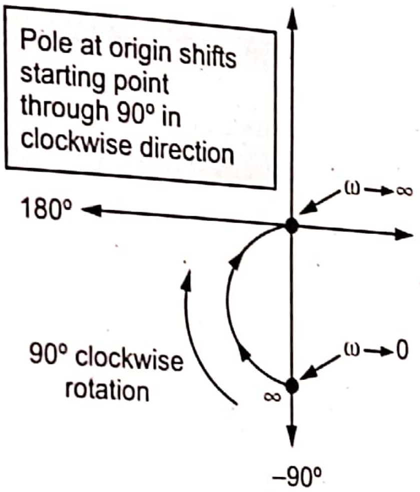

Pole at origin shifts starting point through 90° in clockwise direction

Fig. Q.1.2

Type 2 system : Consider a open loop transfer function

$$G(s)H(s) = \frac{1}{s^2(1+Ts)}$$

$$G(j\omega)H(j\omega) = \frac{1}{j\omega \cdot j\omega \cdot (1+Tj\omega)} = \frac{(1+j0)}{(0+j\omega)(0+j\omega)(1+j\omega T)}$$

$$\therefore \quad M = \frac{1}{\omega^2 \cdot \sqrt{1+T^2\omega^2}}, \quad \phi = -180° - \tan^{-1}\omega T$$

ω	M	ϕ
0	∞	− 180°
∞	0	− 270°

- The rotation of the plot is,

  $-270° - (-180°) = -90°$

  i.e. 90° clockwise.

- So the polar plot is starting from $\infty\angle-180°$ it ends at $0\angle-270°$ with 90° clockwise rotation as shown in the Fig. Q.1.3.

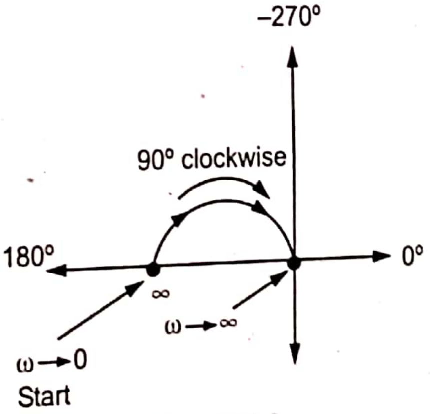


Fig. Q.1.3

6.2 : G.M. and P.M. From Polar and Nyquist Plot

Q.2 How to determine gain margin and phase margin from the Polar plot.

Ans. : According to definition of gain margin,

$$\text{G.M.} = \frac{1}{|G(j\omega)\,H(j\omega)|_{\omega = \omega_{pc}}}$$

- In polar plot $|G(j\omega)\,H(j\omega)|_{\omega = \omega_{pc}}$ is nothing but $l(OQ)$ where Q is the intersection of polar plot with negative real axis. Q is the point corresponding to $\omega = \omega_{pc}$.

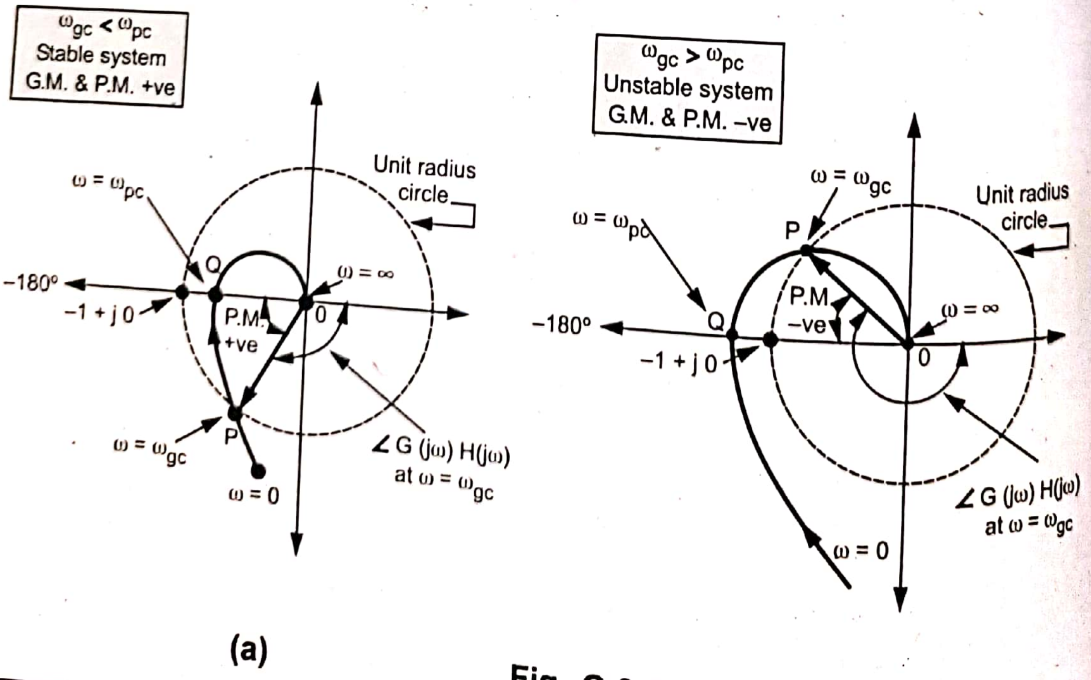
$$\therefore \quad \boxed{\text{G.M.} = \frac{1}{l\,(OQ)}}$$

where Q is intersection of polar plot with negative real axis.

- According to definition of phase margin,

$$\boxed{\text{P.M.} = 180° + \angle G(j\omega)\,H(j\omega)\ |_{\omega = \omega_{gc}}}$$

- Draw the unit radius circle on the polar plot. The point where it intersects polar plot is denoted as P at which $|G(j\omega)\,H(j\omega)| = 1$ and the corresponding frequency i nothing but $\omega = \omega_{gc}$.


ωgc < ωpc
Stable system
G.M. & P.M. +ve
ω = ωpc
Unit radius circle
−180°
Q
ω = ∞
−1 + j 0
P.M.
+ve
0
P
ω = ωgc
ω = 0
∠G (jω) H(jω) at ω = ωgc
ωgc > ωpc
Unstable system
G.M. & P.M. −ve
ω = ωgc
ω = ωpc
P
Unit radius circle
P.M.
−ve
ω = ∞
−180°
Q
−1 + j 0
0
∠G (jω) H(jω) at ω = ωgc
ω = 0

(a)

Fig. Q.2.1

(b)

- So if ϕ is the angle of point P corresponding to $\omega = \omega_{gc}$ the P.M. is the angle subtended by the phasor OP with negative real axis. This is shown in the Fig. Q.2.1.

Important Points to Remember

To find ω_{pc} and intersection of polar plot with negative real axis

i) Rationalize the open loop transfer function G(jω)H(jω).

ii) Separate real and imaginary parts of G(jω)H(jω). Both are the function of ω.

iii) Equate imaginary part to zero to get equation as f(ω) = 0. Solve this to get value of ω which is making this imaginary part zero i.e. $\omega = \omega_{pc}$. This frequency **should be positive finite and greater than zero**. Otherwise it can be concluded that there is no intersection of polar plot with negative real axis.

iv) Substitute this value of ω_{pc} in the real part to get the actual co-ordinates of an intersection of point of polar plot with negative real axis. This point is denoted as Q.

Q.3 Draw the polar plot of the given system $G(s)H(s) = \frac{10}{s(s+2)}$.

☞ **[SPPU : Dec.-15, Marks 9]**

Ans. :

$$G(j\omega)H(j\omega) = \frac{10}{j\omega(2+j\omega)} = M_R \angle \phi_R$$

$$M_R = \frac{10}{\omega\sqrt{4+\omega^2}}, \quad \phi_R = -90^\circ - \tan^{-1}\frac{\omega}{2}$$

Starting ω = 0	$M_R = \infty$,	$\phi_R = -90^\circ$
End ω = ∞	$M_R = 0$,	$\phi_R = -180^\circ$

$$\therefore \text{Rotation of plot} = -180^\circ - (-90^\circ)$$
$$= -90^\circ \text{ clockwise}$$

The polar plot is shown in the Fig. Q.3.1.

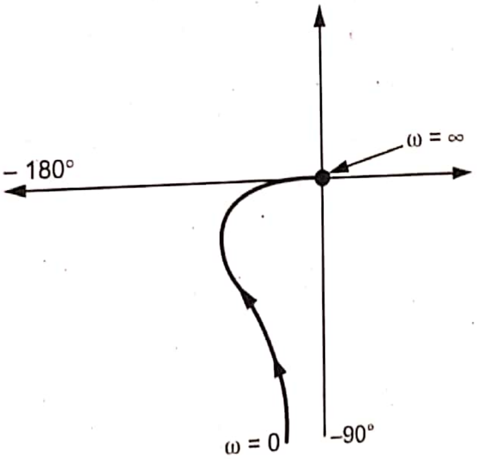


Fig. Q.3.1

Q.4 Draw polar plot for system with open loop transfer function.

$G(s) = \frac{50}{s(s+3)(s+6)}$ **[SPPU : Dec.-17, Marks 8]**

Ans. : The frequency domain T.F. is,

$$G(j\omega) = \frac{50}{j\omega(3+j\omega)(6+j\omega)} = M_R \angle \phi_R$$

$$M_R = \frac{50}{\omega\sqrt{9+\omega^2}\ \sqrt{36+\omega^2}}$$

$$\phi_R = -90^\circ - \tan^{-1}\frac{\omega}{3} - \tan^{-1}\frac{\omega}{6}$$

Start $\omega = 0$	$\infty \angle -90^\circ$	$-270^\circ-(-90^\circ) = -180^\circ$ Clockwise rotation
End $\omega = \infty$	$0 \angle -270^\circ$	

For intersection with real axis, rationalize $G(j\omega)$.

$$G(j\omega) = \frac{50(-j\omega)(3-j\omega)(6-j\omega)}{j\omega(-j\omega)(3+j\omega)(3-j\omega)(6+j\omega)(6-j\omega)}$$

$$= \frac{-450\omega^2 - 50j\omega(18-\omega^2)}{\omega^2(9+\omega^2)(36+\omega^2)}$$

For imaginary part to be zero,

$18 - \omega^2 = 0$

$\therefore \quad \omega^2 = 18$ and $\omega_{pc} = \sqrt{18}$

Using in real part,

$$Q = \frac{-450}{(9+18)(36+18)} = -0.308$$

Thus the polar plot is shown in the Fig. Q.4.1

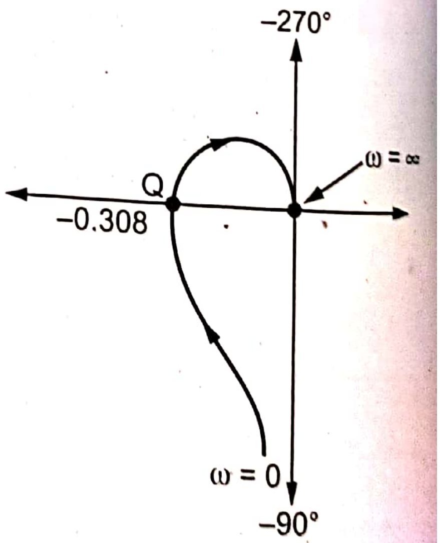


Fig. Q.4.1

Q.5 Draw polar plot of the given system

$G(s)H(s) = \frac{60}{(s+1)(s+2)(s+5)}$

Find the frequency and the magnitude of G(s)H(s) i) at which the plot intersects y axis ii) at which the plot intersects x axis.

[SPPU : May-18, Marks 10]

Ans. : The frequency domain T.F. is,

$$G(j\omega)H(j\omega) = \frac{60}{(1+j\omega)(2+j\omega)(5+j\omega)} = M_R \angle \phi_R$$

$$M_R = \frac{60}{\sqrt{1+\omega^2} \times \sqrt{4+\omega^2} \times \sqrt{25+\omega^2}}$$

$$\phi_R = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{5}$$

Start $\omega = 0$	$6 \angle 0^\circ$	$-270^\circ - 0^\circ = -270^\circ$ Clockwise rotation
End $\omega = \infty$	$0 \angle -270^\circ$	

i) To find Y axis intersection, rationalize $G(j\omega)H(j\omega)$.

$$G(j\omega)H(j\omega) = \frac{60(1-j\omega)(2-j\omega)(5-j\omega)}{(1+j\omega)(1-j\omega)(2+j\omega)(2-j\omega)(5+j\omega)(5-j\omega)}$$

$$= \frac{60[(10-8\omega^2)+j\omega(\omega^2-17)]}{(1+\omega^2)(4+\omega^2)(25+\omega^2)} \quad \text{...(Q.5.1)}$$

For Y intersection, real part is zero.

$$10-8\omega^2 = 0 \quad \text{i.e.} \quad \omega = \mathbf{1.118} \text{ rad./sec}$$

$\therefore$

$|G(j\omega)1+(j\omega)|$ at this $\omega = \mathbf{3.407}$

ii) To find X axis intersection, make imaginary part of equation (Q.5.1) to zero.

$\therefore$ $\omega = \mathbf{0}$ or $\omega = \sqrt{\mathbf{17}}$ are the frequencies at which plot intersects X axis.

... intersects +ve axis

At $\omega = 0$, $|G(j\omega)H(j\omega)| = 6$

At $\omega = \sqrt{17}$, $|G(j\omega)H(j\omega)| = 0.4762$

... intersects –ve axis

The polar plot is shown in the Fig. Q.5.1.

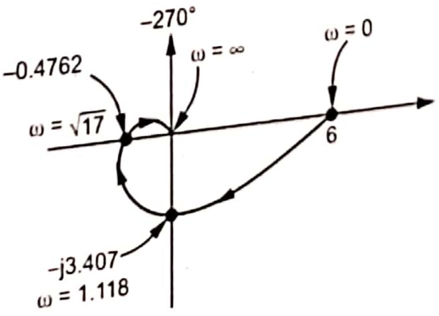


Fig. Q.5.1

6.3 : Nyquist Plot and Nyquist Criterion

Important Points to Remember

- Poles of 1 + G(s)H(s) = Open loop poles of a system.
- Zeros of 1 + G(s)H(s) = Closed loop poles of a system.
- From Nyquist point of view, the system is absolutely stable if all the **zeros of 1 + G(s)H(s).** i.e. closed loop poles of the system are located in the left half of s-plane.

Q.6 What is encirclement ? How to count the number of encirclements of a point by a closed path ?

Ans. : **• A point is said to be encircled by a closed path if it is found to lie inside that closed path.**

- The steps to count the number of encirclements of a point by a closed path are,

1) Draw a vector from a point whose encirclements are to be determined, in such a way to join any point outside that closed path in any direction. Avoid confusing directions.
2) Identify the number of intersections of this vector with a closed path.
3) Mark these intersections with small arrow on the same vector indicating direction of closed path at the time of intersection.
4) Cancel the oppositely directed encirclements. The remaining arrows gives us the number of encirclements of that point.

- **Anticlockwise Encirclements are treated as Positive. Clockwise Encirclements treated as Negative.**
- In the closed path shown in the Fig. Q.6.1 for both the points O and A we get the direction anticlockwise at the point of intersection of the vector drawn with $\tau(s)$ path.
- Hence net number of encirclements denoted as N and +1 i.e. one in anticlockwise direction. Consider a closed path as shown in the Fig. Q.6.2.
- The vector drawn from point A , intersects path at two points. At both the points path is travelling in clockwise direction. Hence total number of encirclements are – 2 i.e. two in clockwise direction.

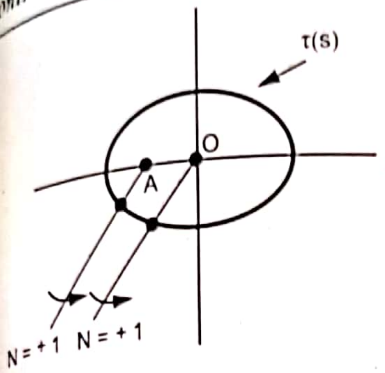


Fig. Q.6.1

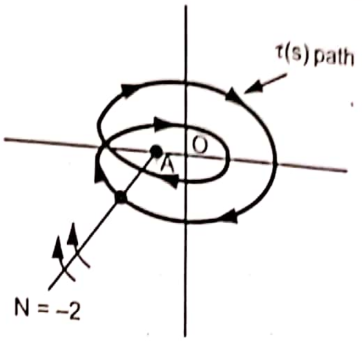


Fig. Q.6.2

Q.7 What is mapping theorem or principle of argument ?

Ans. : • Let F(s) be the single valued function, analytic at all points in s-plane except some finite number of points.

- Consider an arbitrary closed path τ(s) in s-plane in such a way that the function F(s) is analytic at each and every point on τ(s) path i.e. it does not pass through the poles of F(s).
- So the restriction to select a closed path is that it should not pass through the points which are singularities of F(s) i.e. it should not pass through poles of F(s).
- Let P and Z be the number of poles and zeros of F(s) **which are encircled by τ(s) path**. i.e. which are inside the path τ(s). So,

P = Number of poles of F(s) encircled by τ(s).
Z = Number of zeros of F(s) encircled by τ(s).

- According to mapping procedure, the closed path τ(s) in s-plane can be mapped into other plane say F-plane to get a closed path say τ′(s).
- The mapping theorem is actually related to this mapped locus τ′(s) in F-plane.
- Mapping theorem states that the mapped locus τ′(s) encircles the new origin of F-plane as many times as the difference between the number of zeros and poles of F(s) which are encircled by τ(s) path in s - plane mathematically,

$$N = Z - P \text{ where}$$

N = Encirclements of origin of F- plane by $\tau'(s)$ path

P = Number of poles of F(s) encircled by $\tau(s)$ path in s-plane

Z = Number of zeros of F(s) encircled by $\tau(s)$ path in s-plane

- This statement is also called **Principle of Argument.**

Q.8 State and explain Nyquist stability criterion.

[SPPU : Dec.-2000,02,06,16,17, May-04,08,12,15,16,18, Marks 8]

Ans. : Nyquist suggested to select a single valued function F(s) as 1+ G(s)H(s) where G(s)H(s) is open loop transfer function of the system.

$$F(s) = 1 + G(s)H(s)$$

- Poles of G(s)H(s) are the open loop poles which are known as G(s)H(s) is known but zeros of 1 + G(s)H(s) are closed loop poles of the system which are not known. The stability depends on the locations of these zeros of 1 + G(s)H(s) in s-plane.
- To examine whether any of these zeros are located in right half of s-plane or not, Nyquist has suggested to select a $\tau(s)$ path which will encircle the entire right half of s-plane.
- Such a path should start from $s = +j\infty$. It should be continued till $s = -j\infty$ along imaginary axis and should be completed with a semicircle of radius ∞, encircling entire right half of s-plane as shown in the Fig. Q.8.1. This path is called **Nyquist path** and should not be changed except small modifications, while analyzing stability of any system.

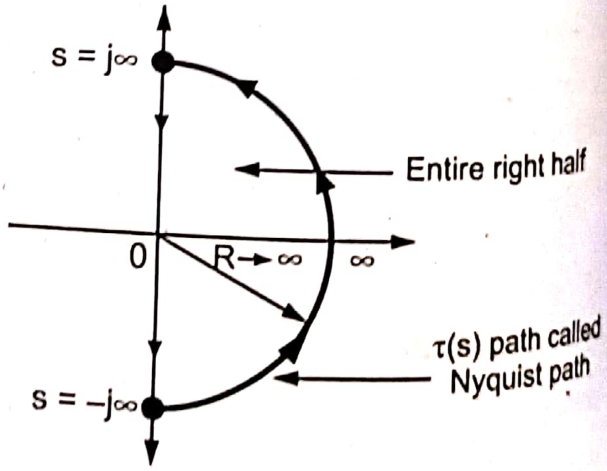


Fig. Q.8.1

- As poles of G(s)H(s) are known which are the poles of 1 + G(s)H(s), we know the value of P i.e. poles of 1 + G(s)H(s) which are encircled by Nyquist path.

- Map all the points on the Nyquist path into F -plane with the help of mapping function 1 + G(s)H(s) to get $\tau'(s)$ locus.
- **This mapped locus obtained in F-plane by mapping all the points on Nyquist path is called Nyquist plot.**
- As this locus is obtained, we can determine the number of encirclements of origin by Nyquist plot in F-plane, say N.
- As per Mapping theorem, N and P must satisfy the equation, N = Z – P and as N and P are known, we can get Z,
- Z = Number of zeros of 1 + G(s)H(s) encircled by Nyquist path in s-plane which are the closed loop poles of the system.
- But as Nyquist path encircles only right half of s-plane, Z = Number of zeros of 1 + G(s)H(s) which are located in right half of s-plane.
- For absolute stability, no zero of 1 + G(s) H(s) must be in right half of s-plane i.e. Z = 0 for stability.
- So Nyquist stability criterion is obtained by substituting Z = 0 in the equation N = Z – P.

$$\boxed{N = -P} \qquad \text{...Nyquist stability criterion}$$

- **Nyquist stability criterion states that for absolute stability of the system, the number of encirclements of new origin of F-plane by Nyquist plot must be equal to number of poles of 1 + G(s)H(s) i.e. poles of G(s)H(s) which are in the right half of s-plane and in clockwise direction.**

Important Points to Remember

Practically while solving the problems, N is the number of encirclements of a critical point $-1+j0$ and not the orgin by Nyqyist plot and the mapping fucntion used in G(s)H(s) rather than 1 + G(s)H(s).

Q.9 Explain the generalised nyquist contour which encloses the entire right half of s-plane. ☞ [SPPU : May-18, Marks 6]

Ans. : • Depending upon the situation of poles of G(s)H(s), Nyquist path should be selected. The guidelines for selection of Nyquist path are given here.

- Let F(s) has two poles on imaginary axis at ± $j\omega_1$ while one pole at origin. Then Nyquist path should be selected as shown in the Fig. Q.9.1.

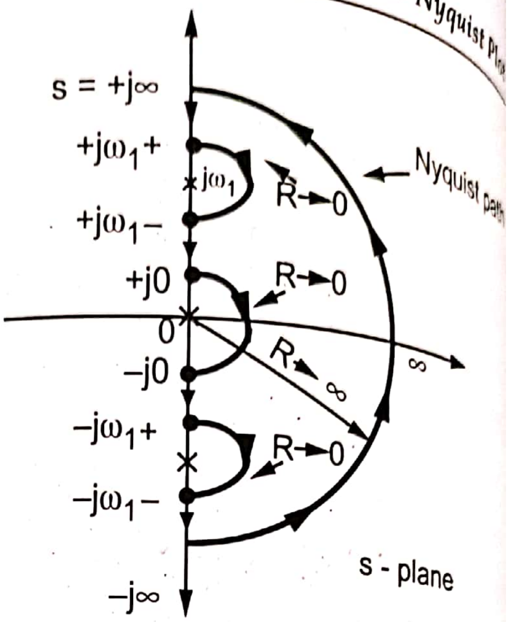


Fig. Q.9.1 Generalised Nyquist Contour

- The points by which path is modified are :

$+j\omega_1^+$ →A point just above + $j\omega_1$, very very close to + $j\omega_1$.

$+j\omega_1^-$ →A point which is very very close to + $j\omega_1$, but just below it.

$-j\omega_1^+$ →A point which is very very close to – $j\omega_1$, but just above it.

$-j\omega_1^-$ →A point which is very very close to – $j\omega_1$ but just below it.

$+j0$→A point which is very very close to origin but just above it on positive imaginary axis hence denoted as + j0

$-j0$→A point which is very very close to origin but just below it on negative imaginary axis hence denoted as – j0.

The various sections of Nyquist path are,

Section	Start	End	Comment
I	$s = +j\infty$	$s = +j\omega_1^+$	Along the imaginary axis.
II	$s = +j\omega_1^+$	$s = +j\omega_1^-$	Along a semicircle of radius tending to zero.
III	$s = +j\omega_1^-$	$s = +j0$	Along the imaginary axis.
IV	$s = +j0$	$s = -j0$	Along semicircle of radius tending to zero.
V	$s = -j0$	$s = -j\omega_1^+$	Along the imaginary axis.
VI	$s = -j\omega_1^+$	$s = -j\omega_1^-$	Along semicircle of radius tending to zero.
VII	$s = -j\omega_1^-$	$s = -j\infty$	Along the imaginary axis.
VIII	$s = -j\infty$	$s = +j\infty$	Along the semicircle of $R \rightarrow \infty$ encircling entire right half.

- The mapping of these sections in F-plane can be achieved by drawing the polar plots for various sections by shortcut method.
- After mapping all sections, we will get a closed plot called **Nyquist plot**.
- The section V, VI and VII are mirror images of section III, II and I respectively. Hence their mapping in F-plane will be also mirror images **about real axis.**

Q.10 List the steps used to solve the problem using Nyquist criterion. ☞ **[SPPU : May-16, Marks 6]**

Ans. :

Step 1 : Count how many number of poles of G(s)H(s) are in the right half of s-plane i.e. with positive real part. This is the value of P.

Step 2 : Decide the stability criterion as N = – P i.e. how many times Nyquist plot should encircle '–1 +j0' point for absolute stability.

Step 3 : Select Nyquist path as per the function G(s)H(s).

Step 4 : Analyse the sections as starting point and terminating point of plot. Last section analysis is not required.

Step 5 : Mathematically find out ω_{pc} and intersection of Nyquist plot with negative real axis by rationalizing $G(j\omega)H(j\omega)$. This point of intersection is denoted as Q.

Step 6 : With the knowledge of step 4 and 5, sketch the Nyquist plot.

Step 7 : Count the number of encirclements N of $-1 + j0$ by Nyquist plot. If this matches with the criterion decided in step 2 system is stable, otherwise unstable.

$$\text{G.M.} = \frac{1}{|OQ|} \quad \text{where}$$

Q = Intersection point of Nyquist plot with negative real axis obtained in step 5.

$$\boxed{\text{G.M.} = 20 \operatorname{Log}_{10} \frac{1}{|OQ|} \text{ dB}}$$

Q.11 For a certain control system $G(s)H(s) = \frac{K}{s(s+2)(s+10)}$. Sketch the Nyquist plot and hence calculate the range of values of K for stability.

☞[SPPU : Dec.-02, 03, 11, May-06, Marks 8]

Ans. :

Step 1 : P = 0

Step 2 : N = − P = 0, the critical point −1 + j0 should not get encircled by Nyquist plot.

Step 3 : Pole at origin hence Nyquist path is as shown in the Fig. Q.11.1.

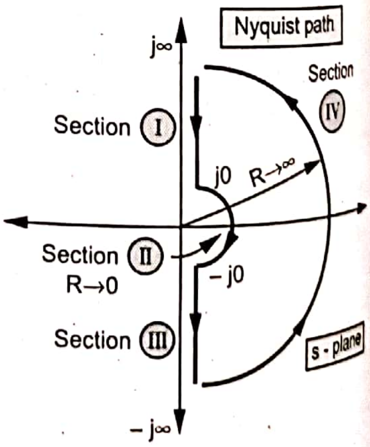


Fig. Q.11.1

Step 4 :

$$G(j\omega)H(j\omega) = \frac{K}{j\omega(2+j\omega)(10+j\omega)}$$

$$M = |G(j\omega)H(j\omega)|$$

$$= \frac{K}{\omega \times \sqrt{4+\omega^2} \times \sqrt{100+\omega^2}}$$

$$\phi = \frac{\tan^{-1}\left(\frac{0}{K}\right)}{\tan^{-1}\left(\frac{\omega}{0}\right)\tan^{-1}\left(\frac{\omega}{2}\right)\tan^{-1}\left(\frac{\omega}{10}\right)}$$

$$= -90^\circ - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{10}$$

Section I : $s = +j\infty$ to $s = +j0$ i.e. $\omega \to \infty$ to $\omega \to +0$

Starting point	$\omega \to \infty$	$0 \angle -270^\circ$	$-90^\circ - (-270)^\circ = +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \to +0$	$0 \angle +90^\circ$	

Section II : $s = +j0$ to $s = -j0$ i.e. $\omega \to +0$ to $\omega \to -0$

Starting point	$\omega \to +0$	$\infty \angle -90^\circ$	$90^\circ - (-90)^\circ = +180^\circ$ Anticlockwise rotation
Terminating point	$\omega \to -0$	$\infty \angle +90^\circ$	

Section III is mirror image of section about real axis.

Section IV is an origin.

Step 5 : $G(j\omega)H(j\omega) = \dfrac{K}{j\omega(10+j\omega)(2+j\omega)}$

Rationalizing ,

$$G(j\omega)H(j\omega) = \frac{K(-j\omega)(10-j\omega)(2-j\omega)}{(j\omega)(-j\omega)(10+j\omega)(10-j\omega)(2+j\omega)(2-j\omega)}$$

$$\therefore \quad G(j\omega)H(j\omega) = \frac{-Kj\omega[20-12j\omega-\omega^2]}{\omega^2(4+\omega^2)(100+\omega^2)}$$

$$= \frac{-12K\omega^2}{D} - \frac{Kj\omega(20-\omega^2)}{D}$$

where $D = \omega^2(4+\omega^2)(100+\omega^2)$

Equating imaginary part to zero,

$$\omega(20-\omega^2) = 0 \quad \text{i.e.} \quad \omega^2 = 20 \quad \text{i.e.} \quad \omega_{pc} = \sqrt{20}$$

Substituting in real part,

$$\text{Point Q} = \frac{-12\,K \times 20}{20 \times (20+4) \times (100+20)} = -\frac{K}{240}$$

Step 6 : The Nyquist plot is,

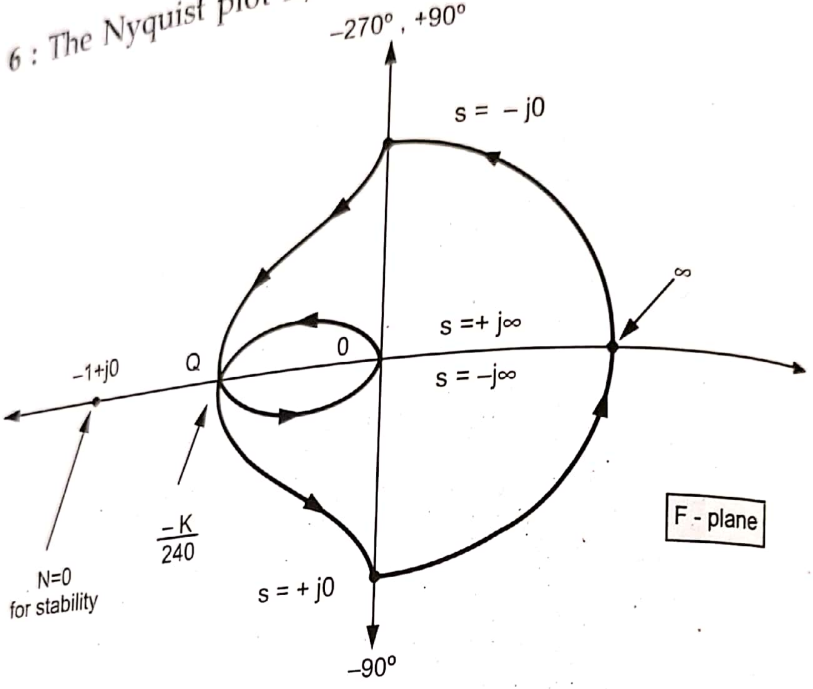


Fig. Q.11.2

Step 7 : Now for absolute stability, N = 0

i.e. it should be located on left side of point Q i.e. $|OQ| < 1$

$$\therefore \quad \left| -\frac{K}{240} \right| < 1$$

$$\therefore \quad K < 240$$

So range of values of K for stability is

$$\boxed{0 < K < 240}$$

Q.12 Draw Nyquist plot for following open loop system $G(s) = \frac{50}{s\,(s+3)\,(s+6)}$ and check for stability.

☞ [SPPU : May-15, Marks 9]

Ans. : Refer Q.8 for the procedure. In this example K = 50 and three poles.

By rationalizing, $\omega_{pc} = \sqrt{18}$ and $Q = -0.3086$

The Nyquist plot is shown in the Fig. 5. As N = 0, the system is stable in nature as there is no open loop pole in right half of s-plane.

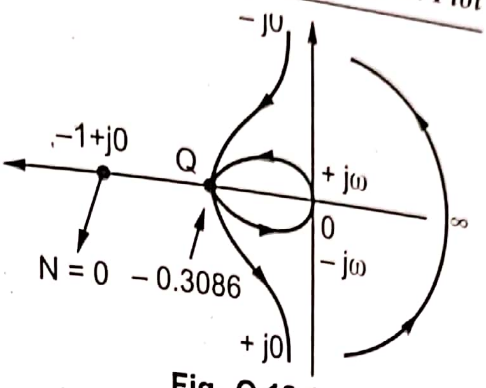


Fig. Q.12.1

Q.13 A control system with closed loop transfer function. $G(s)H(s) = \frac{K(s-1)}{s(s+1)}$. Plot Nyquist plot and comment on stability.

☞ **[SPPU : Dec.-16, Marks 10]**

Ans. :

Step - 1 : No open loop poles in right half of s-plane hence P = 0.

Step - 2 : For closed loop stability, N = − P = 0

Step - 3 : Nyquist path is shown in the Fig. Q.13.1.

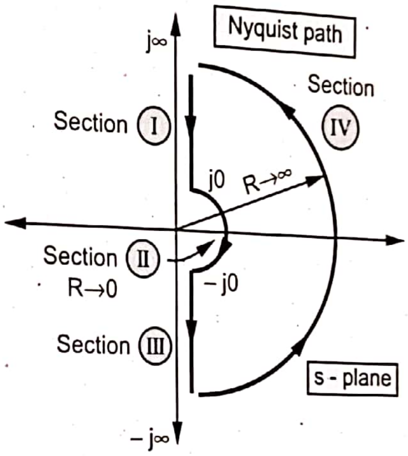


Fig. Q.13.1

Step-4 : $G(j\omega)H(j\omega) = \frac{K(-1+j\omega)}{j\omega\,(1+j\omega)}$

Section I : s = j∞ to s = +j0

Starting ω → ∞	$0\angle\frac{90^\circ}{90^\circ\ 90^\circ} = 0\angle -90^\circ$	$90^\circ - (-90^\circ) = +180^\circ$ Anticlockwise
Terminating ω → +0	$\infty\angle\frac{180^\circ}{90^\circ\ 0^\circ} = \infty\angle +90^\circ$	

Section II : s = +j0 to s = −j 0

Starting ω → + 0	$\infty\angle +90^\circ$	$270^\circ - 90^\circ = 180^\circ$ Anticlockwise
Terminating ω → − 0	$\infty\angle\frac{180^\circ}{-90^\circ\ 0^\circ} = \infty\angle 270^\circ$	

Section III : Minor image of section I about real axis.

Section IV : Not required for closed loop stability.

Step 5 : Intersection with real axis.

$$G(j\omega)H(j\omega) = \frac{K(-1+j\omega)(-j\omega)(1-j\omega)}{(j\omega)(-j\omega)(1+j\omega)(1-j\omega)} \quad \text{... Rationalize}$$

$$= \frac{+2\omega^2 K - j\omega(-1+\omega^2)}{\omega^2(1+\omega^2)}$$

$\therefore \quad \omega_{pc} = 1$ to make imaginary part zero

$$\therefore \quad Q = \left.\frac{+2\omega^2 K}{\omega^2(1+\omega^2)}\right|_{\omega=1} = +K$$

Step 6 : The Nyquist plot is shown in Fig. Q.13.2.

Step 7 : N = +1 which is not matching with criteria in step 2.

Hence system is **unstable** for any positive value of K.

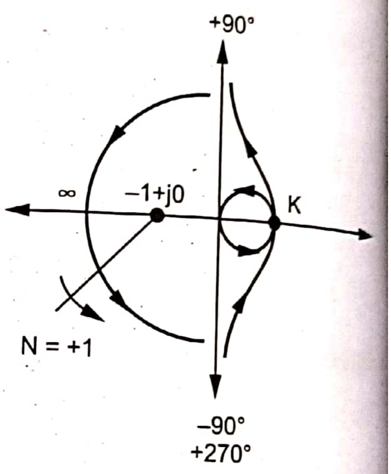


Fig. Q.13.2

Q.14 Sketch Nyquist plot for the system with open loop transfer function given by GH(s) = $\frac{20}{(s+2)(s+3)}$ comment on stability of system.

☞ **[SPPU : May-17, Marks 12]**

Ans. :

Step - 1 : P = 0, no open loop pole in right half s-plane.

Step - 2 : N = −P = 0 for closed loop stability.

Step - 3 : Nyquist path is shown in the Fig. Q.14.1.

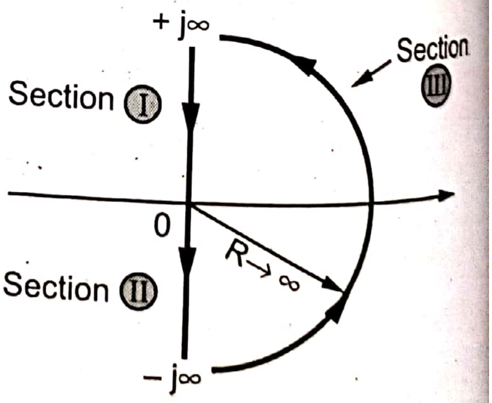


Fig. Q.14.1

Step - 4 : $G(j\omega)H(j\omega) = \frac{20}{(2+j\omega)(3+j\omega)}$

Section I : s = +j∞ to s = 0

Starting $\omega \to \infty$	$0\angle\frac{0^\circ}{90^\circ\ 90^\circ} = 0\angle-180^\circ$	$0^\circ-(-180^\circ)$ $= +180^\circ$ Anticlockwise
Terminating $\omega = 0$	$3.33\angle 0^\circ$	

Section II : Minor image of section I about real axis.

Section III : Not required for closed loop stability.

Step - 5 : Intersection with real axis.

$$G(j\omega)H(j\omega) = \frac{20\,(2-j\omega)(3-j\omega)}{(2+j\omega)\,(2-j\omega)(3+j\omega)(3-j\omega)} = \frac{20(6-\omega^2)-100\,j\omega}{(4+\omega^2)(9+\omega^2)}$$

For imaginary part to be zero, $\omega = 0$ and $|G(j\omega)H(j\omega)|$ is 3.333 at $\omega = 0$. There is no intersection of Nyquist plot with negative real axis.

Step - 6 : Nyquist plot is shown in Fig. Q.14.2.

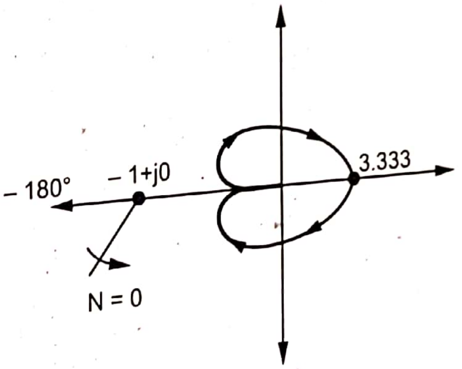


Fig. Q.14.2

Step 7 : As N = 0, it matches criteria in step 2.

Hence system is **stable** in nature.

Important Points to Remember

Behaviour of Right Half Pole of G(s)H(s)

- If there is a pole located in right half of s-plane in G(s)H(s) then it is important to know its contribution in deciding the resultant phase angle.
- Let the pole is $\frac{1}{(s-a)}$ then in frequency domain it is $\frac{1}{(j\omega-a)}$

and

as $\omega \to +\infty$, $\angle\frac{1}{(j\omega-a)} = +90^\circ$ and

as $\omega \to +0$ or -0, $\angle\frac{1}{(j\omega-a)} = +180^\circ$

Q.18 Consider the following open loop system

$$G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$$

Draw Nyquist plot and comment on the stability of closed **system.** ☞ **[SPPU : May-18, Marks**

Ans. :

Step - 1 : P = 1, one open pole of G(s)H(s) in right half s-plan

Step - 2 : N = −P = −1, once clockwise encirclement of −1 + j0 stability.

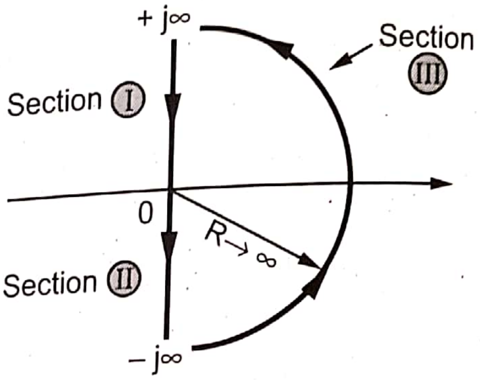


Fig. Q.15.1

Step - 3 : The Nyquist path is as shown in the Fig. Q.15.1.

Step - 4 :

$$G(j\omega)H(j\omega) = \frac{(2+j\omega)}{(1+j\omega)(-1+j\omega)}$$

Section I : s = +j∞ to s = 0

Starting $\omega \to \infty$	$0\angle\frac{90^\circ}{90^\circ\ 90^\circ} = 0\angle-90^\circ$	$-180^\circ-(-90^\circ)$ $= +90^\circ$ Clockwise
End $\omega = 0$	$2\angle\frac{0^\circ}{0^\circ\ 180^\circ} = 2\angle-180^\circ$	

Section II : Mirror image of section I about real axis.

Section III : Not required for closed loop stability.

Step 5 : Intersection with real axis.

Rationalize G(jω) H(jω),

$$G(j\omega)H(j\omega) = \frac{(2+j\omega)(1-j\omega)(-1-j\omega)}{(1+j\omega)(1-j\omega)(-1+j\omega)(-1-j\omega)}$$

$$= \frac{-2(1+\omega^2)-j\omega(\omega^2+1)}{(1+\omega^2)(1+\omega^2)}$$

For imaginary part to be zero, $\omega = 0$ and other values are imaginary hence not valid.

$$G(j\omega)H(j\omega) = -2+j0 = Q \text{ point at } \omega = 0.$$

Step - 6 : The Nyquist plot is shown in Fig. Q.15.2.

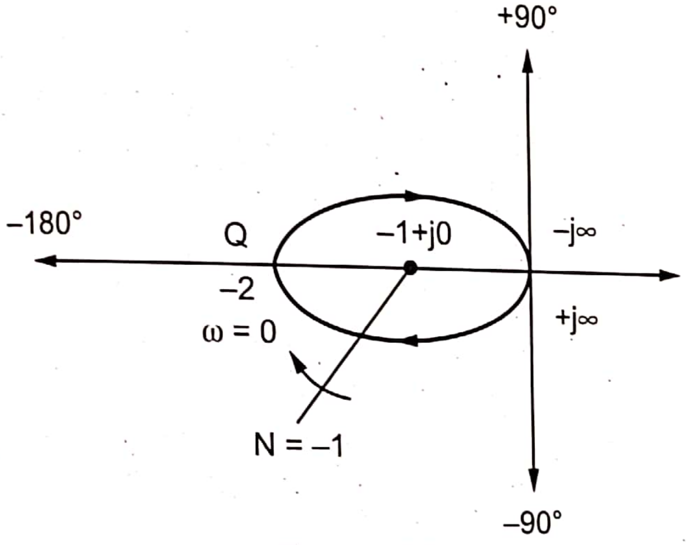
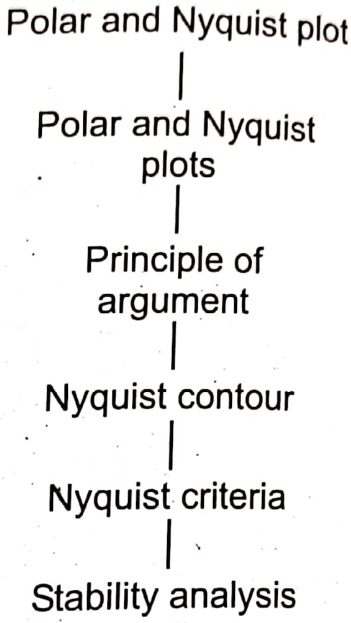


Fig. Q.15.2

Step - 7 : Nyquist plot encircles − 1 + j0 once in clockwise direction hence P = −1. This matches the criteria in step 2, hence system is stable.

Memory Map :


Polar and Nyquist plot
Polar and Nyquist plots
Principle of argument
Nyquist contour
Nyquist criteria
Stability analysis

UNIT - 6

7 PID Controllers and Control System Components

Important Points to Remember

- The controller is an element which accepts the error in some form and decided the proper corrective action. The output of the controller is applied to the process to be controlled. The controller is a heart of a control system.
- The various types of controllers used are proportional (P), Integral (I), Derivative (D), Proportional + Integral (PI), Proportional + Derivative (PD) and Proportional + Integral + Derivative (PID).

7.1 : Proportional Controller

Q.1 Explain the proportional controller stating its characteristics.

☞ **[SPPU : Dec.-05, 09, 17, May-06, 09, 15, 16, 17, 18, Marks 4]**

Ans. : Proportional controller (P) : In this control mode, the output of the controller is simple proportional to the error e(t).

- The relation between the error e(t) and the controller output p is determined by constant called **proportional gain constant** denoted as K_p. The output of the controller is a linear function of the error e(t).
- Though there exists linear relation between controller output and the error, for a zero error the controller output should not be zero, otherwise the process will come to halt. Hence there exists some controller output p_o for the zero error.
- Hence mathematically the proportional control mode is expressed as,

$$\boxed{p(t) = K_p e(t) + p_0}$$

where K_p = Proportional gain con

p_0 = Controller output with zero error.

Characteristics :

1. When the error is zero, the controller output is constant equal p_0.
2. If the error occurs, then for every 1 % of error the correction of K_p% is achieved. If error is positive, K_p % correction ge added to p_0 and if error is negative, K_p% correction ge subtracted from p_0.
3. The band of error exists for which the output of the controller between 0 % to 100 % without saturation.
4. The gain K_p and the error band PB are inversely proportional each other.
5. It produces an **offset error** in the output.

7.2 : Integral Controller

Q.2 Explain the integral controller stating its characteristics.

☞ [SPPU : Dec.-05, 09, May-06, 10, 18, Marks 4]

Ans. : • The controller which is based on the history of error called integral controller.

- In such a controller, the value of the controller output p(t) changed at a rate which is proportional to the actuating error signal e(t).
- Mathematically it is expressed as,

$$\boxed{\frac{d\,p(t)}{dt} = K_i e(t)}$$

where

K_i = Constant relating error and rate

$$\boxed{p(t) = K_p e(t) + p_0}$$ ... (1)

where K_p = Proportional gain constant.

p_0 = Controller output with zero error.

Characteristics :

1. When the error is zero, the controller output is constant equal to p_0.
2. If the error occurs, then for every 1 % of error the correction of K_p% is achieved. If error is positive, K_p % correction gets added to p_0 and if error is negative, K_p% correction gets subtracted from p_0.
3. The band of error exists for which the output of the controller is between 0 % to 100 % without saturation.
4. The gain K_p and the error band PB are inversely proportional to each other.
5. It produces an **offset error** in the output.

7.2 : Integral Controller

2.2 Explain the integral controller stating its characteristics.

☞ **[SPPU : Dec.-05, 09, May-06, 10, 18, Marks 4]**

ns. : • The controller which is based on the history of error is alled integral controller.

In such a controller, the value of the controller output p(t) is changed at a rate which is proportional to the actuating error signal e(t).

Mathematically it is expressed as,

$$\boxed{\frac{d\,p(t)}{dt} = K_i e(t)}$$

nere K_i = Constant relating error and rate

- The constant K_i is also called integral constant. Integrating the above equation, actual controller output at any time t can be obtained as,

$$\boxed{p(t) = K_i \int e(t)dt + p(0)} \quad \dots (1)$$

where $p(0)$ = Controller output when integral action starts i.e. at $t = 0$.

- The output signal from the controller, at any instant is the area under the actuating error signal curve up to that instant. If the value of the error is doubled, the value of p(t) varies twice as fast i.e. rate of the controller output change also doubles.
- If the error is zero, the controller output is not changed.

Characteristics

1. If error is zero, the output remains at a fixed value equal to what is was, when the error became zero.
2. If the error is not zero, then the output begins to increase or decrease, at a rate K_i% per second for every ± 1 % of error.

- In pure integral mode, error can oscillate about zero and can be cyclic. Hence in practice integral mode is never used alone but combined with the proportional mode, to enjoy the advantages of both the modes.

7.3 : Derivative Controller

Q.3 Explain the derivative controller stating its characteristics.

☞ [SPPU : Dec.-05, 09, May-06, 12, 18, Marks 4]

Ans. : • In this mode, the output of the controller depends on the time rate of change of the actual errors. Hence it is also called **rate action mode** or **anticipatory action mode**.

- The mathematical equation for the mode is,

$$\boxed{p(t) = K_d \frac{d\,e(t)}{dt}}$$ where K_d = Derivative gain constant.

- The derivative gain constant indicates by how much % the controller output must change for every % per sec rate of change of the error. Generally K_d is expressed in minutes.
- The important feature of this type of control mode is that for a given rate of change of error signal, there is a unique value of the controller output.
- The advantage of the derivative control action is that it responds to the rate of change of error and can produce the significant correction before the magnitude of the actuating error becomes too large.
- Derivative control thus anticipates the actuating error, initiates an early corrective action and tends to increase stability of the system improving the transient response.

Characteristics

1. For a given rate of change of error signal, there is a unique value of the controller output.
2. When the error is zero, the controller output is zero.
3. When the error is constant i.e. rate of change of error is zero, the controller output is zero.
4. When error is changing, the controller output changes by K_d % for even 1 % per second rate of change of error.

7.4 : Proportional + Integral Controller (PI)

Q.4 Explain the features of PI controller.

☞ **[SPPU : May-15, 16, 17, Dec.-15, 17, Marks 4]**

Ans. : • This is a composite control mode obtained by combining the proportional mode and the integral mode.

- The mathematical expression for such a composite control is,

$$\boxed{p(t) = K_p\, e(t) + K_p K_i \int_0^t e(t)dt + p(0)}$$

where p(0) = Initial value of the output at t = 0

- The important advantage of this control is that one to one correspondence of proportional mode is available while the offset gets eliminated due to integral mode.

Characteristics :

1. When the error is zero, the controller output is fixed at the value that integral mode had when the error went to zero. This is nothing but p(0).
2. When the error is not zero, proportional mode adds the correction while the integral term starts increasing or decreasing from its initial value depending upon reverse or direct action.
3. It improves the steady state accuracy.
4. It increases the rise time so response becomes slow.
5. It decreases bandwidth of the system.
6. It filters out the high frequency noise.
7. It makes the response more oscillatory.

- PI mode can be used in the systems with the frequent or large load changes.
- The Fig. Q.4.1 shows the block diagram of PI controller.

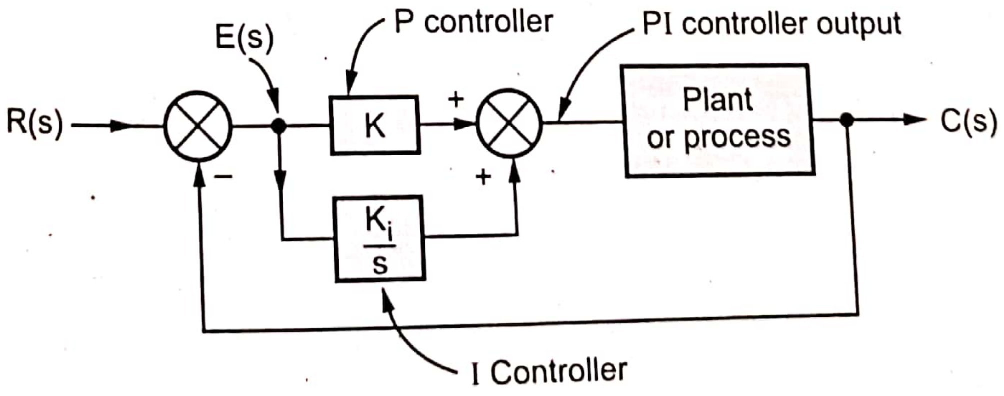


Fig. Q.4.1 PI controller

7.5 : Proportional + Derivative Controller (PD)

Q.5 Explain the features of PD controller.

☞ [SPPU : May-10, 12, Dec.-05, 09, 15, 18, Marks 4]

Ans. : The series combination of proportional and derivative control modes gives proportional plus derivative control mode.

- The mathematical expression for the PD composite control is,

$$\boxed{p(t) = K_p e(t) + K_p K_d \frac{d\,e(t)}{dt} + p(0)}$$

Characteristics :

1. It improves the damping and reduces overshoot.
2. It reduces the rise time and makes response fast.
3. It makes the response stable very fast.
4. It improves the bandwidth of the system.
5. It can not eliminate offset error.
6. It may make the noise dominant at high frequencies.
7. It is not very effective for lightly damped systems.
8. It may require a relatively large capacitor while the circuit implementation.

- The Fig. Q.5.1 shows the block diagram of PD controller.

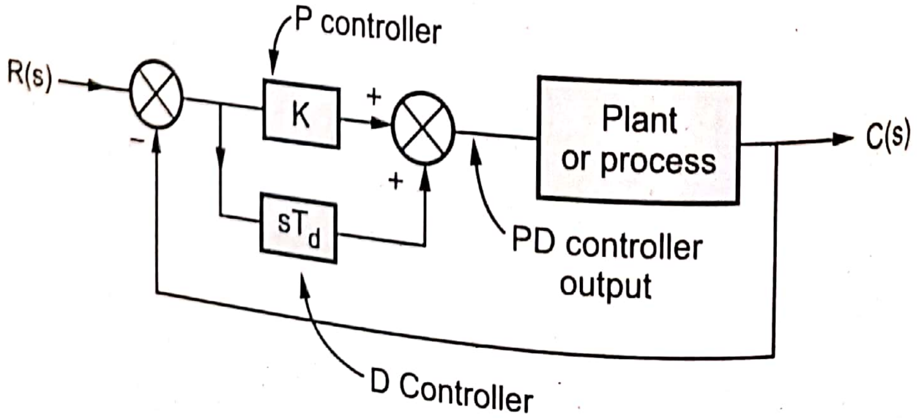


Fig. Q.5.1 PD controller

7.6 PID Controller

Explain the PID controller and its features.

[SPPU : Dec.-05,07,09,10,11,16,17, May-06,10,11,12,15,17,18, Marks 6]

Ans. : • The composite controller including the combination of the roportional, integral and derivative control mode is called PID ntrol mode and the controller is called three mode controller.

- It is very much complex to design but very powerful in action.
- Mathematically such a control mode can be expressed as,

$$\boxed{p(t) = K_p\, e(t) + K_p K_i \int_0^t e(t)dt + K_p K_d \frac{d\,e(t)}{dt} + p(0)}$$

here $\quad p(0) = $ Initial value of the output

- The Fig. Q.6.1 shows the block diagram of PID controller.

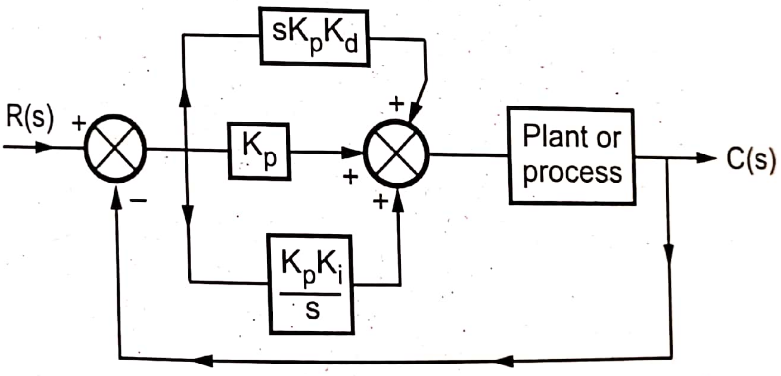


Fig. Q.6.1

- This mode has advantages of all the modes. The integral mode eliminates the offset error of the proportional mode and the response is also very fast due to derivative mode. The sudden response is produced due to derivative mode. Thus it can be used for any process condition.
- With the PID control action, there is no offset, no oscillations with least settling time. So there is improvement in both transient as well as steady state response.
- The Fig. Q.6.2 shows the response of PID control for a particular error signal, assuming direct action.

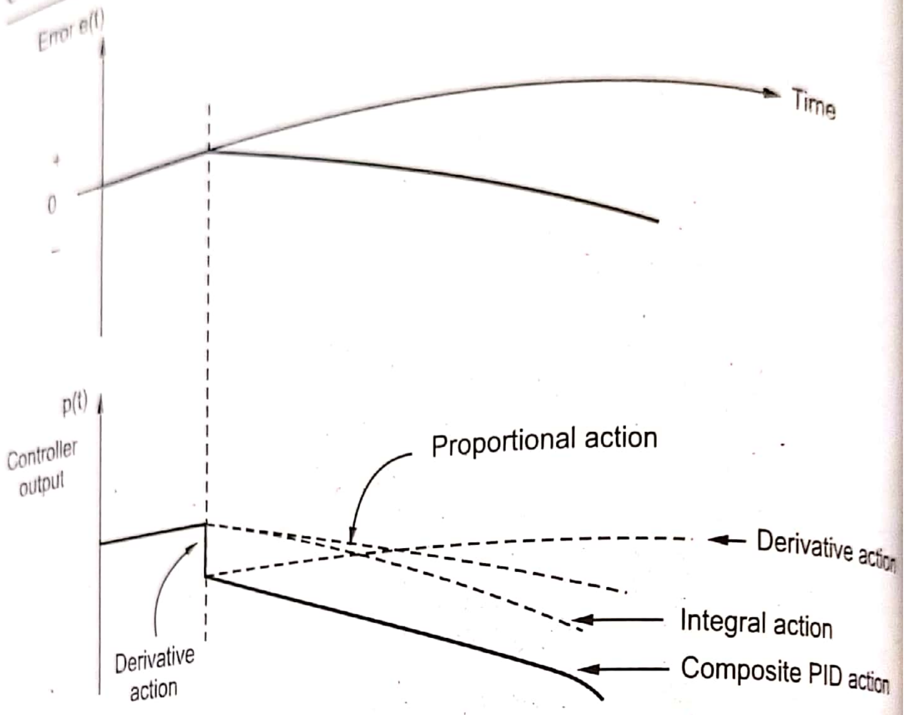


Fig. Q.6.2

- The proportional and PD control produces the offset error. It requires significant time to attain the steady state.
- The PI control eliminates the offset but at the expense of higher maximum overshoot, a long period of oscillations and more settling time.
- The PD control produces the steady state very quickly with least oscillations and smallest maximum overshoot but offset is significant.
- With PID control, there is no offset and system achieves the steady state with less settling time. Thus PID is the ultimate process composite controller.

7.7 : Rate Feedback Controller

Q.7 Explain the rate feedback controller.

Ans. : • This controller uses the feedback which is derivative of output signal internally and compares with signal proportional to the error.

- It is also called output derivative controller.
- The Fig. Q.7.1 shows the use of rate feedback controller with standard second order system.

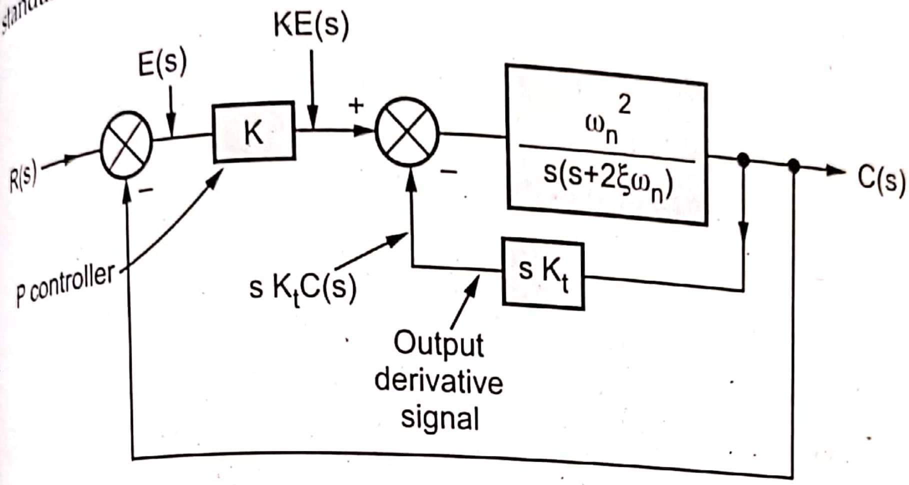

KE(s)
E(s)
+
R(s)
K
−
ω_n^2
$s(s+2\xi\omega_n)$
−
C(s)
P controller
s K_tC(s)
s K_t
Output
derivative
signal

Fig. Q.7.1 Rate feedback controller

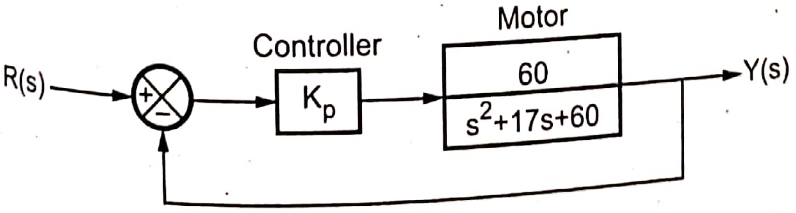
Due to this controller,

1. Damping ratio gets improved.
2. Reduces settling time and rise time.
3. It affects the steady state.
4. The natural frequency is unaffected.
5. It makes the response fast.

- But due to added zero, it is necessary to consider stability of the system while designing this controller.

7.8 : Solved Problems on Controllers

Q.8 The system given below is so design of have damping ratio 0.707. Determine the required values of K_p for the given damping ratio.
☞ [SPPU : Dec.-16, Marks 8]


Motor
Controller
R(s)
+
−
K_p
60
$s^2+17s+60$
Y(s)

Ans. : $\xi = 0.707$

The closed loop T.F. with K_p is,

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{60\,K_p}{s^2+17s+60}}{1+\dfrac{60\,K_p}{s^2+17s+60}} = \frac{60\,K_p}{s^2+17s+60(K_p+1)}$$

Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$,

$$\omega_n^2 = 60(K_p+1) \quad \text{i.e.} \quad \omega_n = \sqrt{60(K_p+1)}$$

$$2\xi\omega_n = 17 \quad \text{i.e.} \quad \xi = \frac{17}{2\omega_n}$$

$$\therefore \quad 0.707 = \frac{17}{2\sqrt{60(K_p+1)}}$$

$$\therefore \quad \sqrt{60(K_p+1)} = 12.0226 \quad \text{... Squaring and solving}$$

$$K_p = \mathbf{1.409}$$

Q.9 A second order system has an open loop transfer function $G(s) = \dfrac{K}{s(0.2s+1)}$ where K is proportional gain. Find the value of K so that steady state error will be < 0.5 when input is $\dfrac{\pi}{6}t$.

☞ **[SPPU : Dec.-17, Marks 8]**

Ans. : System is type 1 system.

Input is $\frac{\pi}{6}t$ i.e. ramp with $A = \frac{\pi}{6}$.

$$\therefore \quad K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0}\frac{sK}{s(0.2s+1)} = K$$

$$e_{ss} = \frac{A}{K_v} = \frac{\pi}{6K} \quad \text{but} \quad e_{ss} < 0.5$$

$$\therefore \quad \frac{\pi}{6K} < 0.5 \quad \text{i.e.} \quad K > \frac{\pi}{6\times 0.5} > 1.047$$

Thus minimum value of K = 1.047

Q.10 A unity feedback system has the plant transfer function $G(s) = \dfrac{C(s)}{M(s)} = \dfrac{10}{s(s+2)}$. A proportional plus derivative control is employed to control the dynamics of the system. Determine

i) The damping factor and undamped natural frequency when $K_d = 0$

ii) The value of K_d such that damping factor is 0.6.

☞ [SPPU : May-17, Marks 8]

Ans. : i) When $K_d = 0$ then,

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{10}{s(s+2)}}{1+\frac{10}{s(s+2)}} = \frac{10}{s^2+2s+10}$$

Comparing denominator with $s^2 + 2\xi\omega_n s + \omega_n^2$,

$\omega_n^2 = 10$, $\omega_n = \sqrt{10}$, $2\xi\omega_n = 2$, $\xi = \mathbf{0.316}$

ii) With K_d, system is as shown in the Fig. Q.10.1.

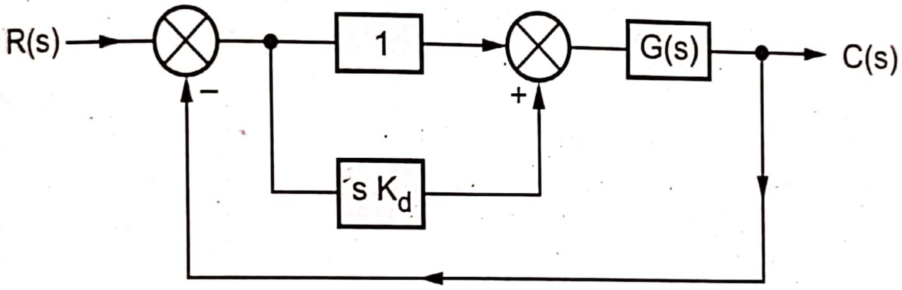


Fig. Q.10.1

$$\therefore \quad G(s) = \frac{(1+sK_d)10}{s(s+2)}, \quad H(s) = 1$$

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\frac{(1+sK_d)10}{s(s+2)}}{1+\frac{(1+sK_d)10}{s(s+2)}} = \frac{10(1+sK_d)}{s^2+s(2+10K_d)+10}$$

$\therefore$ $\omega_n^2 = 10$, $\omega_n = \sqrt{10}$, $2\xi\omega_n = 2+10K_d$

$$\therefore \quad \xi = \frac{2+10K_d}{2\sqrt{10}} \text{ but } \xi = 0.6 \text{ (given)}$$

$$\therefore \quad 0.6 = \frac{2+10K_d}{2\sqrt{10}} \quad \text{i.e.} \quad K_d = \mathbf{0.1794}$$

Q.11 A feedback system which employs output rate feedback is shown. Determine the derivative feedback constant K_h which will increase the damping factor of the system to 0.6. What is the steady state error to unit ramp input. **☞ [SPPU : Dec.-15, Marks 8]**

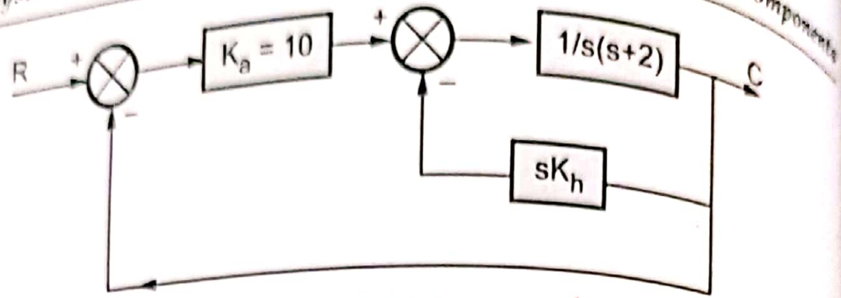


Fig. Q.11.1

Ans. : Reducing minor loop,

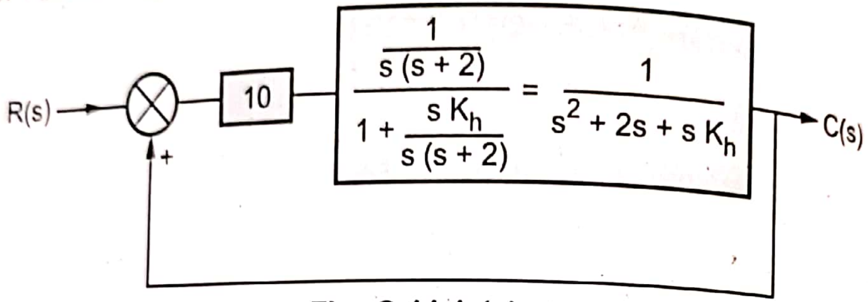

R(s)
+
10
$\frac{\frac{1}{s(s+2)}}{1+\frac{sK_h}{s(s+2)}} = \frac{1}{s^2+2s+sK_h}$
C(s)

Fig. Q.11.1 (a)

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{\dfrac{10}{s^2+2s+sK_h}}{1+\dfrac{10}{s^2+2s+sK_h}} = \frac{10}{s^2+s(2+K_h)+10}$$

Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$,

$$\omega_n^2 = 10,\ \omega_n = \sqrt{10},\ 2\xi\omega_n = 2+K_h,\ \xi = \frac{2+K_h}{2\sqrt{10}}$$

But $\quad \xi = 0.6,\ 0.6 = \dfrac{2+K_h}{2\sqrt{10}} \quad$ i.e. $\quad K_h = \mathbf{1.7947}$

From Fig. 6(a), $\quad G(s) = \dfrac{10}{s(s+2+K_h)},\ H(s) = 1$

For unit ramp input, $K_v = \lim\limits_{s\to 0} sG(s)H(s) = \dfrac{10}{2+K_h}$

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{1}{\left(\dfrac{10}{2+K_h}\right)} = \frac{2+K_h}{10}$$

$$\therefore \quad e_{ss} = \frac{2+1.7947}{10} = \mathbf{0.37947} \quad \text{...For designed } K_h$$

7.9 : Design of PID Controller in Time Domain

Q.12 Explain the procedure to design PID controller in time domain with root locus.

☞ **[SPPU : May-16, Marks 8]**

Ans. : • The root locus is the powerful tool which is used for the time domain design of P, PI and PID controllers.

- If we consider a location of a complex pole in s-plane as shown in the Fig. Q.12.1 then angle made by line OP with negative real axis is $\cos^{-1}\xi$, the imaginary part of the pole is ω_d while OP indicates the value of ω_n.
- Thus if ω_n and ξ is known then the location of dominant closed loop can be obtained.

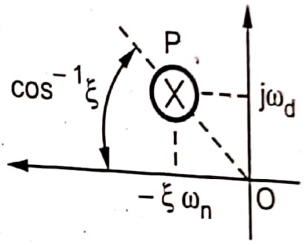


Fig. Q.12.1

- The root locus helps to position such dominant poles at the required locations by varying the proportional gain.
- Depending upon the specifications, the controllers can be designed by placing the poles and zeros at specific locations.
- A PID controller has two zeros and a pole at origin. In a root locus, the branches approach to zeros hence selecting the controller gains properly, the zeros can be placed so as to modify the root locus which matches the required specifications.
- Due to two zeros and one pole of PID controller, one asymptote is less in the new root locus and the angles of asymptotes and centroid location also change accordingly.
- The new root locus then can be adjusted so as to achieve the required specifications by selecting the controller gains.
- Consider a PI controller with a transfer function $G_c(s) = K_p + \frac{K_i}{s}$. Its pole-zero configuration is shown in the Fig. Q.12.2.
- With a PI controller, a type 0 system is converted to type 1, type 1 is converted to type 2 and K_p does not affect the steady state

error as error for type 1 system for step input is always zero while error for type 2 system for ramp input is always zero.

- Thus a method of designing a PI controller is to select a zero $s = -\frac{K_i}{K_p}$ so that it is relatively close to the origin and away from the most significant poles of the plant. By observing its effect on the root locus, it can then be adjusted so as to achieve required goal.

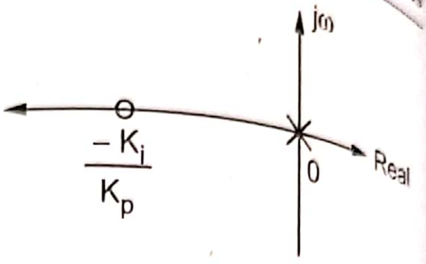


Fig. Q.12.2

7.10 : Zeigler-Nicholas Method of Tuning Controller

Q.13 Explain the Zeigler-Nicholas method of tuning the PID controller. **☞ [SPPU : May-15, 16, 17, Dec.-15, 16, 17, Marks 6]**

Ans. : Finding suitable values of the constants K_p, T_i and K_d which makes the performance of a PID controller stable, optimum, robust and fast is called tuning of a controller.

Zeigler-Nicholas method uses following steps to tune the controller :

Step 1 : Bring the process as close as the specified set point manually.

Step 2 : Turn the PID controller in P mode with $T_i = \infty$ and $K_d = 0$. Increase K_p such that overall closed loop is in a continuous oscillations.

Step 3 : The value of K_p for which system shows sustained oscillations is called critical or ultimate gain denoted as K_u. The time between two successive peaks in the continuously oscillating output is called ultimate time period of oscillations denoted as T_u.

Step 4 : The table is provided by Ziegler-Nichols which gives results to design the various constants of controllers based on the values of K_u and T_u. Thus depending upon the type of controller P, PI or PID, using the values in the Table Q.13.1, the controller can be tuned.

Controller type	K_p	T_i	K_d
P	$0.5\ K_u$	∞	0
PI	$0.45\ K_u$	$\frac{T_u}{1.2} = 0.833\ T_u$	0
PID	$0.6\ K_u$	$\frac{T_u}{2} = 0.5\ T_u$	$\frac{T_u}{8} = 0.125\ T_u$

Table Q.13.1

Hence the T.F. of PID controller is,

$$G_c(s) = K_p + \frac{K_p}{T_i}\frac{1}{s} + K_p K_d s$$

Using the Table Q.4.1, the final $G_c(s)$ is,

$$\therefore \quad \boxed{G_c(s) = \frac{0.075 K_u T_u \left(s+\frac{4}{T_u}\right)^2}{s}}$$

Q.14 Design a PID controller for system with unity feedback and $G(S) = \frac{K}{(s+3)(s^2+s+1)}$.

☞ [SPPU : May-16, Marks 8]

Ans.: Let K is the proportional controller gain. The characteristic equation is

$1 + G(s)H(s) = 0$

$$\therefore 1 + \frac{K}{(s+3)(s^2+s+1)} = 0 \quad \text{i.e.} \quad s^3 + 4s^2 + 4s + K + 3 = 0$$

Routh's array is,

$$\begin{array}{c|cc} s^3 & 1 & 4 \\ s^2 & 4 & K+3 \\ s^1 & \frac{13-K}{4} & 0 \\ s^0 & K+3 & \end{array}$$

For critical value of K,

$13 - K = 0$

$\therefore K_u = K = 13$

$A(s) = 4s^2 + K + 3 = 0$

$\therefore s^2 = \frac{-(K+3)}{4} = -4$

$\therefore s = \pm j2$

$\therefore$ Frequency of sustained oscillations = 2 rad/s

But $\omega = 2\pi f = \frac{2\pi}{T_u}$ i.e. $T_u = \frac{2\pi}{2} = 3.1416$

Hence according to Ziegler - Nichols method,

$K_p = 0.6\ K_u = 0.6 \times 13 = 7.8,\ T_i = 0.5\ T_u = \mathbf{1.5708}$

$K_d = \frac{T_u}{8} = 0.3927$

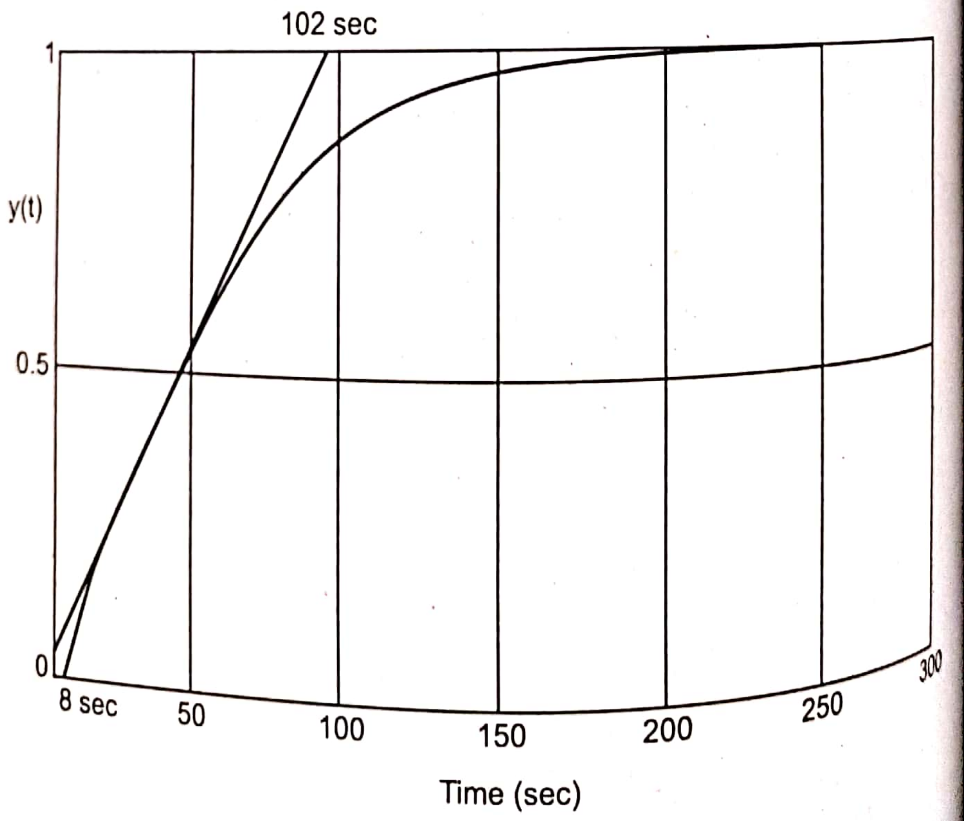
Hence the T.F. of the PID controller is,

$$\therefore G_c(s) = \frac{0.075\ K_u\ T_u \left(s + \frac{4}{T_u}\right)^2}{s} = \boxed{\frac{3.063\ (s + 1.2732)^2}{s}}$$

Q.15 An open loop test of a temperature control system yields the reaction curve shown below. The system open loop transfer function is given by

$$G(s) = \frac{1}{(20s + 1)(50s + 1)}$$

Used Ziegler Nichols method to determine K_p, K_I, K_d ? For a quarter step response PID control system. ☞ **[SPPU : Dec.-16, Marks 8]**



Ans. : From the given quarter step response.

$$L = 8, \quad T = 102 - 8 = 94$$

For PID control

$$K_p = \frac{1.2\,T}{L} = \frac{1.2 \times 94}{8} = 14.1$$

$$T_i = 2L = 16$$

$$K_d = 0.5\,L = 4$$

$$\therefore \quad G_c(s) = \frac{0.6\,T}{s}\left(s + \frac{1}{L}\right)^2 = \frac{0.6 \times 94}{s}\left(s + \frac{1}{8}\right)^2$$

$$\therefore G_c(s) = \boxed{\frac{56.4(s+0.125)^2}{s}}$$

Thus, $K_p = \mathbf{14.1}$, $K_i = \frac{1}{T_i} = \mathbf{0.0625}$, $K_d = \mathbf{4}$

Q.16 Using Ziegler Nichols method design a PID controller for system with open loop transfer function $G(s) = \frac{15}{s(s+1)(s+3)}$. Write close loop transfer function of plant including PID controller.

☞ [SPPU : May-17, 18, Marks 8]

Ans. : Let K is proportional controller gain.

$$\therefore \quad G(s) = \frac{15\,K}{s(s+1)(s+3)}, \quad H(s) = 1$$

The characteristic equation is 1 + G(s)H(s) = 0

$$\therefore \quad 1 + \frac{15K}{s(s+1)(s+3)} = 0 \quad \text{i.e.} \quad s^3 + 4s^2 + 3s + 15K = 0$$

Routh's array is,

$$\begin{array}{c|cc} s^3 & 1 & 3 \\ s^2 & 4 & 15K \\ s^1 & \frac{12-15K}{4} & 0 \\ s^0 & 15K & \end{array}$$

From s^1 row.

$$12 - 15K = 0$$

$$\therefore \quad K = K_u = 0.8$$

$$A(s) = 4s^2 + 15K = 0$$

$$\therefore \quad s^2 = -3$$

$$\therefore \quad s = \pm j\,1.732$$

∴ Frequency of sustained oscillations = 1.732

But

$$\omega = 2\pi f = \frac{2\pi}{T_u} = 1.732$$

∴

$$T_u = 3.627$$

According to Ziegler - Nichols method,

$$K_p = 0.6\,K_u = 0.48, \quad T_i = 0.5\,T_u = 1.8135$$

$$K_d = \frac{T_u}{8} = 0.4533$$

$$\therefore G_c(s) = \frac{0.75\,K_u T_u\left(s+\frac{4}{T_u}\right)^2}{s} = \boxed{\frac{0.217(s+1.1028)^2}{s}}$$

Hence closed loop T.F. is

$$\therefore \frac{C(s)}{R(s)} = \frac{G_c(s)G(s)}{1+G_c(s)G(s)} = \boxed{\frac{3.255s^2+7.18s+3.958}{s^4+4s^3+6.255s^2+7.178s+3.958}}$$

7.11 : Lead and Lag Compensators

Important Points to Remember

- In practice, if a system is to be redesigned so as to meet the required specifications, it is necessary to alter the system by adding an external device to it. Such an alteration of system using an additional suitable device is called compensation of system and the device is called compensator.
- The two types of compensating networks are lead network and lag network.

Q.17 Explain lag and lead network with their transfer function.

☞ [SPPU : Dec.-17, Marks 4]

OR What is lead network ? Obtain the TF of lead network.

☞ [SPPU : May-05, 16, April-15, 17 (Insem), Marks 4]

Ans. : Lead network : This network produces the output with phase lead with respect to input.

The network is shown in the Fig. Q.17.1.

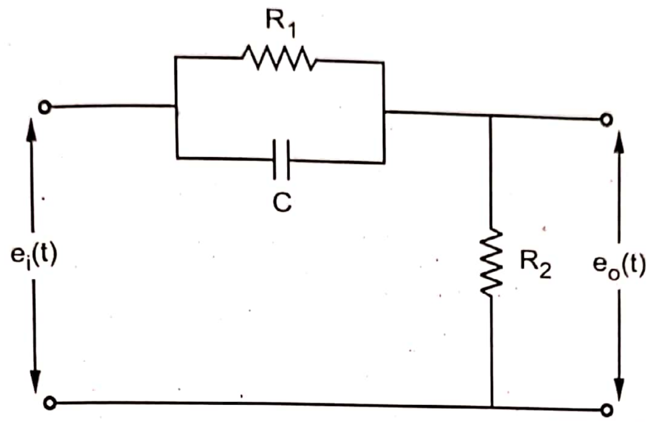
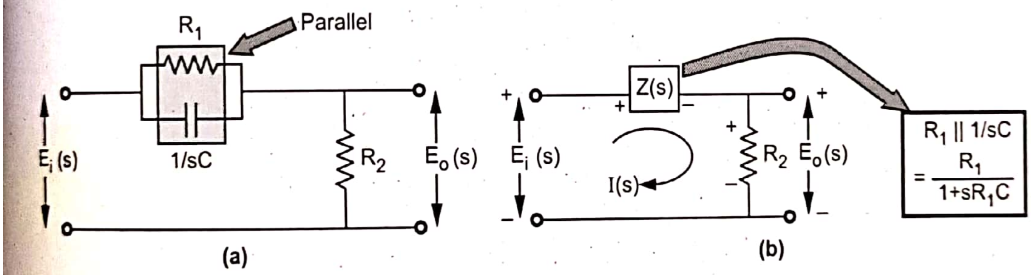

R_1
C
$e_i(t)$
R_2
$e_o(t)$

Fig. Q.17.1 Lead network

- Draw the s-domain network of the given network replacing R_1 by R_1 and C by (1/sC) with all variables replaced by Laplace variables.


R_1
Parallel
1/sC
$E_i(s)$
R_2
$E_o(s)$
Z(s)
I(s)
$R_1 \parallel 1/sC = \frac{R_1}{1+sR_1C}$

(a)

(b)

Fig. Q.17.2

- Applying KVL to the loop,

$$-I(s)\,Z(s) - I(s)\,R_2 + E_i(s) = 0 \text{ and } I(s)\,R_2 = E_o(s)$$

$$\therefore \quad I(s) = \frac{E_i(s)}{Z(s)+R_2} \text{ and } \frac{E_i(s)}{Z(s)+R_2} \times R_2 = E_o(s)$$

$$\therefore \frac{E_o(s)}{E_i(s)} = \frac{R_2}{Z(s)+R_2} = \frac{R_2}{\frac{R}{1+sRC}+R_2} = \boxed{\frac{sR_1R_2C+R_2}{R_2+R_1+sR_1R_2C}}$$

- The pole-zero locations are shown in the Fig. Q.17.3.

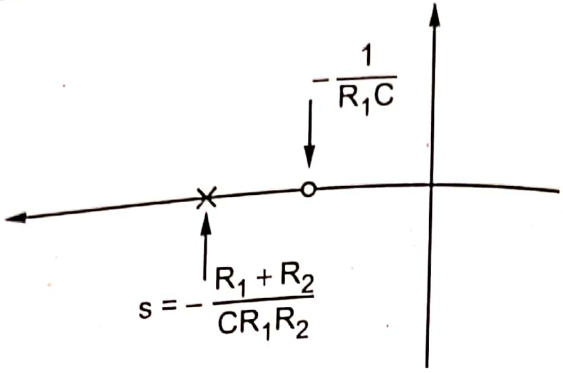
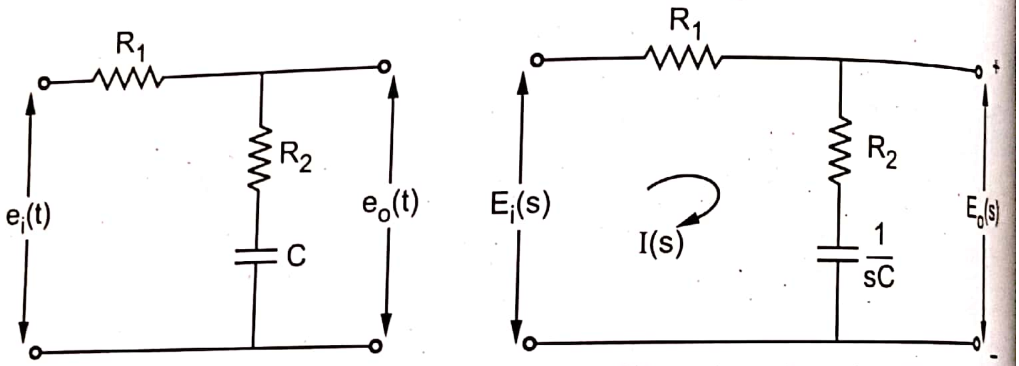


Fig. Q.17.3

Lag network : This network add lagging phase angle to the output with respect to input.

- The network is shown in the Fig. Q.17.4 (a) while its s-domain network is shown in the Fig. Q.17.4 (b).



(a) Lag network

(b) s - domain network

Fig. Q.17.4

$$\therefore \quad I(s) = \frac{E_i(s)}{R_1+R_2+\frac{1}{sC}} = \frac{sCE_i(s)}{1+sC(R_1+R_2)}$$

$$\therefore \quad E_o(s) = I(s)\times\left(R_2+\frac{1}{sC}\right) = \frac{sCE_i(s)}{1+sC(R_1+R_2)}\times\frac{1+sR_2C}{sC}$$

$$\therefore \quad \frac{E_o(s)}{E_i(s)} = \boxed{\frac{1+sR_2C}{1+sC(R_1+R_2)}}$$

- The pole-zero locations are shown in the Fig. Q.17.5.

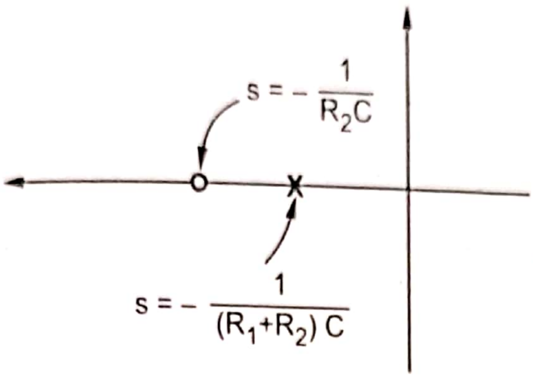


Fig. Q.17.5

7.12 : Potentiometer

Q.18 What is potentiometer ? Explain its role as an error detector. Obtain its transfer function. ☞ [SPPU : April-15 (Insem) , Marks 4]

Ans. : • It is an electromechanical transducer that converts mechanical energy into an electrical energy. The input to the device is in the form of a mechanical displacement either linear or rotational.

- When voltage is applied across the fixed terminals of the potentiometer, the output voltage, which is measured across the variable terminals, to proportional to the input displacement either linear or according to some nonlinear relation.
- Since the voltage across the variable terminal and reference is proportional to the shaft displacement of the potentiometer, when a voltage is applied across fixed terminals, the device can be used to indicate the absolute position of a system.
- Fig. Q.18.1 shows arrangement when the housing of potentiometer is fixed at reference, the output voltage e(t) will be proportional to the shaft position $\theta_c(t)$, in case of a rotary motion then

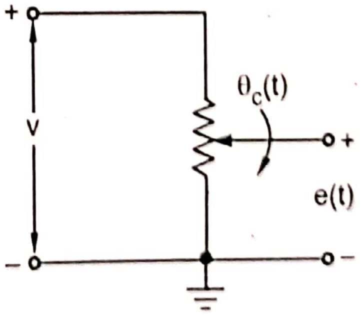


Fig. Q.18.1

$$\therefore \quad \boxed{e(t) = K_s\,\theta_c(t)}$$

where K_s is the proportionality constant.

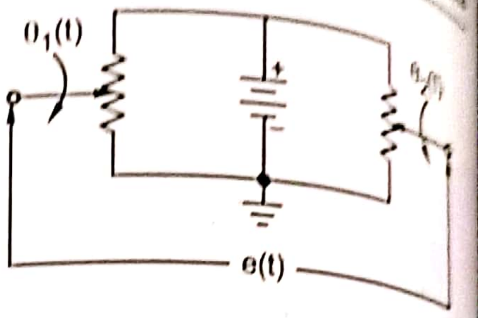
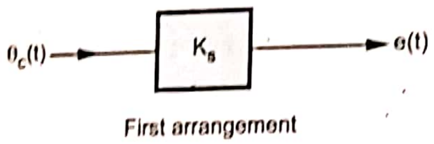


Fig. Q.18.2

- Consider another arrangement as shown in Fig. Q.18.2. This is more flexible and is obtained by using two potentiometers connected in parallel.
- This type allows the comparison of two remotely located shaft positions. The output voltage is taken across the variable terminals of the two potentiometers.

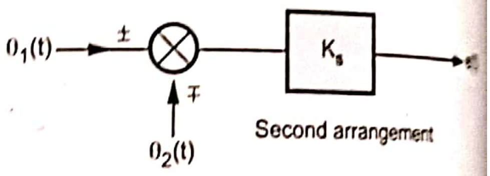
$$\boxed{\text{Output } e(t) = K_s\,[\theta_1(t) - \theta_2(t)]}$$

- Block diagram representation, of above two setups are as shown in Fig. Q.18.2 and Fig. Q.18.4.



First arrangement

Fig. Q.18.3



Second arrangement

Fig. Q.18.4

Thus the T.F. of the potentiometer is

$$\frac{E(s)}{\theta_C(s)} = \frac{E(s)}{\theta_1(s)-\theta_2(s)} = K$$

Potentiometer as an Error Detector

- To study the use of potentiometer as error detector consider the automatic tank level control system as shown in the Fig. Q.18.5.
- This control system can maintain the liquid level h of the tank within accurate tolerance of the desired liquid level even though the output flow rate through the value V_1 is varied.

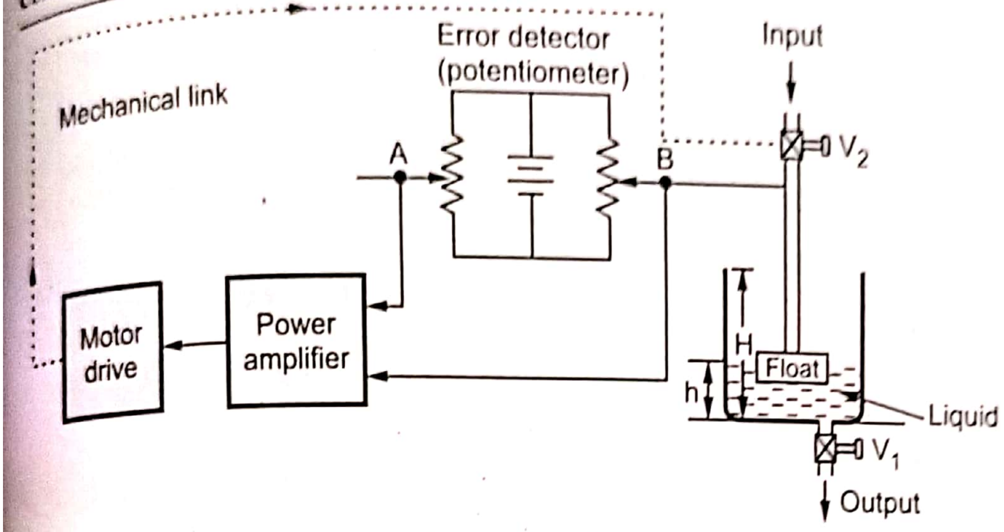


Fig. Q.18.5

- The liquid level is sensed by float, which positions the slider B on a potentiometer. The slider arm A of another potentiometer is positioned corresponding to desired liquid level H. When liquid level rises or falls, the potentiometers gives error voltage proportional to the change in liquid level.
- The error voltage actuates the motor through a power amplifier which in turn controls the point in order to restore the desired liquid level.
- Thus the control system automatically attempts to correct any deviation between the actual and desired levels in the tank.

7.13 : A.C. and D.C. Servomotors

Important Points to Remember

- The servosystem is the one in which the output is some mechanical variable like position, velocity or acceleration.
- The motors used in such servosystems are called servomotors, which are used to convert electrical signal applied into angular movement of the shaft.
- These motors have linear relationship between electrical control signal and the output over a wide range.
- These are classified as a.c. servomotors and d.c. servomotors.

- The d.c. servomotors are further classified as field controlled d.c. motor and armature controlled motor.

Q.19 Derive the transfer function of a.c. servomotor and obtain its block diagram representation. ☞ **[SPPU : April-15, 17(Insem), Marks 9]**

Ans. : • The various **approximations** to derive transfer function are

i) A servomotor rarely operates at high speeds. Hence for a given value of control voltage, T ∝ N characteristics are perfectly linear.

ii) In order that T ∝ N characteristics are directly proportional to voltage applied to its control phase, we assume T ∝ N characteristics are straight lines and equally spaced.

- Torque at any speed 'N' is,

$$T_m = K_{tm} E_2 + m\frac{d\theta_m}{dt} \quad \text{...(1)}$$

where $\frac{d\theta_m}{dt}$ is speed of motor, E_2 is voltage applied to control winding

- If load consists inertia J_m and friction B_m we can write,

$$T_m(s) = J_m s^2 \theta_m + B_m s \theta_m \quad \text{...(2)}$$

- The Laplace transform of equation (1) is

$$T_m(s) = K_{tm} E_2(s) + m s \theta_m(s) \quad \text{...(3)}$$

- Equating equations (2) and (3),

$$\therefore \; K_{tm} E_2(s) + m s \theta_m(s) = J_m s^2 \theta_m(s) + B_m s \theta_m(s)$$

$$\therefore \; \frac{\theta_m(s)}{E_2(s)} = \frac{K_{tm}}{s(s J_m - m + B_m)}$$

$$= \frac{K_{tm}}{s(B_m - m)\left[1 + \frac{s J_m}{(B_m - m)}\right]}$$

$$\therefore \; \boxed{\frac{\theta_m(s)}{E_2(s)} = \frac{K_m}{s(1 + \tau_m s)}}$$

where $$K_m = \frac{K_{tm}}{B_m - m}, \quad \text{and} \quad \tau_m = \frac{J_m}{B_m - m}$$

- The block diagram of a.c. servomotor is shown in the Fig. Q.19.1.

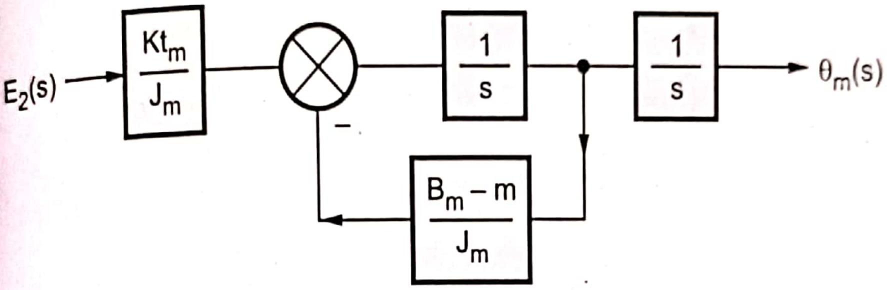

$E_2(s)$
$\frac{Kt_m}{J_m}$
–
$\frac{1}{s}$
$\frac{1}{s}$
$\theta_m(s)$
$\frac{B_m - m}{J_m}$

Fig. Q.19.1 Block diagram of a.c. servomotor

Q.20 Derive the T.F. of armature controlled d.c. servomotor.

☞ **[SPPU : April-15, 16 (Insem), May-16, 17, 18, Marks 6]**

Ans. : • The armature controlled d.c. motor is shown in the Fig. Q.20.1.

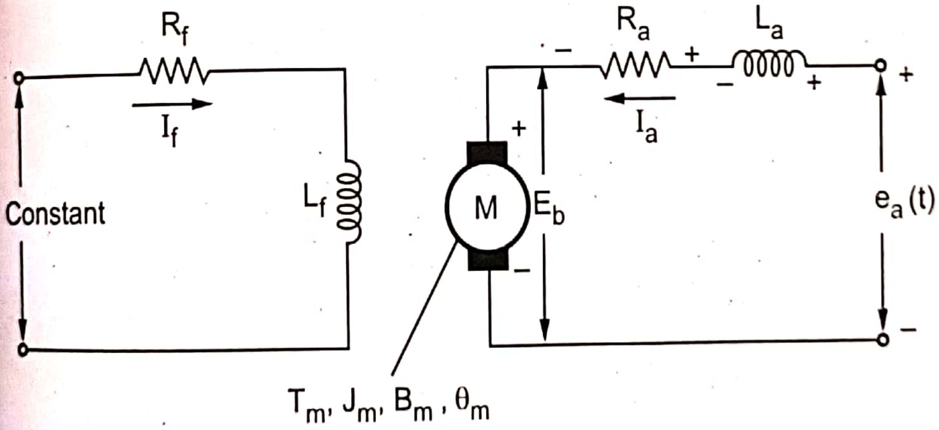

R_f
I_f
Constant
L_f
R_a
L_a
–
+
–
+
I_a
+
M
E_b
–
+
$e_a(t)$
–
T_m, J_m, B_m, θ_m

Fig. Q.20.1

Assumptions :

i) Flux is directly proportional to current through field winding, hence $\phi_m = K_f I_f$ = Constant.

ii) Torque produced is proportional to product of flux and armature current.

- Hence, $T = K'_m \phi I_a$ i.e. $T = K'_m K_f I_f I_a$

iii) Back e.m.f. is directly proportional to shaft velocity ω_m, as flux ϕ is constant.

- As $\omega_m = \frac{d\theta(t)}{dt}$, $E_b = K_b\,\omega_m(s) = K_b\,s\,\theta_m(s)$

Apply Kirchhoff's law to armature circuit :

$$e_a = E_b + I_a(R_a) + L_a\frac{di_a}{dt}$$

- Take Laplace transform,

$$\therefore \quad E_a(s) = E_b(s) + I_a(s)[R_a + s\,L_a] \quad \text{i.e.} \quad I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + s\,L_a}$$

$$I_a(s) = \frac{E_a(s) - K_b\,s\,\theta_m(s)}{R_a + s\,L_a}$$

Now $$T_m = K'_m\,K_f\,I_f\,I_a$$

$$= K'_m\,K_f\,I_f\left\{\frac{E_a(s) - K_b\,s\,\theta_m(s)}{R_a + s\,L_a}\right\}$$

- The torque is used to drive the load with inertia J_m and aga the friction with constant B_m hence we can write,

$$T_m = \{J_m\,s^2 + s\,B_m\}\theta_m(s)$$

- Equating equations of T_m,

$$\frac{K'_m\,K_f\,I_f\,E_a(s)}{(R_a + s\,L_a)} = \frac{K'_m\,K_f\,I_f\,K_b\,s\,\theta_m(s)}{(R_a + s\,L_a)} + (J_m\,s^2 + s\,B_m)\theta_m(s)$$

$$\therefore \quad \frac{K'_m\,K_f\,I_f}{(R_a + s\,L_a)}E_a(s) = \left[\frac{K'_m\,K_f\,I_f\,K_b\,s}{(R_a + s\,L_a)} + J_m s^2 + s\,B_m\right]\theta_m(s)$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{s\,R_a\,B_m\,(1 + s\tau_m)(1 + s\,\tau_a)}}{1 + \dfrac{K_m \cdot sK_b}{s\,R_a\,B_m\,(1 + s\,\tau_m)(1 + s\,\tau_a)}} = \frac{G(s)}{1 + G(s)H(s)}}$$

where $\tau_m = J_m/B_m$, $\tau_a = \frac{L_a}{R_a}$ and $K_m = K'_m\,K_f$

$$G(s) = \frac{K_m}{s\,R_a\,B_m\,(1 + s\,\tau_m)(1 + s\,\tau_a)} \quad \text{and } H(s) = sK_b$$

Therefore T.F. can be represented in its block diagram form as i Fig. Q.20.2.

- As $\omega_m = \frac{d\theta(t)}{dt}$, $E_b = K_b \, \omega_m(s) = K_b \, s \, \theta_m(s)$

Apply Kirchhoff's law to armature circuit :

$$e_a = E_b + I_a \, (R_a) + L_a \frac{di_a}{dt}$$

- Take Laplace transform,

$\therefore$ $E_a(s) = E_b(s) + I_a(s)[R_a + s\,L_a]$ i.e. $I_a(s) = \frac{E_a(s) - E_b(s)}{R_a + s\,L_a}$

$$I_a(s) = \frac{E_a(s) - K_b \, s\,\theta_m(s)}{R_a + s\,L_a}$$

Now $T_m = K'_m \, K_f \, I_f \, I_a$

$$= K'_m \, K_f \, I_f \left\{ \frac{E_a(s) - K_b \, s\,\theta_m(s)}{R_a + s\,L_a} \right\}$$

- The torque is used to drive the load with inertia J_m and against the friction with constant B_m hence we can write,

$$T_m = \{J_m \, s^2 + s\,B_m\}\theta_m(s)$$

- Equating equations of T_m,

$$\frac{K'_m \, K_f \, I_f \, E_a(s)}{(R_a + s\,L_a)} = \frac{K'_m \, K_f \, I_f \, K_b \, s\,\theta_m(s)}{(R_a + s\,L_a)} + (J_m \, s^2 + s\,B_m)\theta_m(s)$$

$$\therefore \quad \frac{K'_m \, K_f \, I_f}{(R_a + s\,L_a)} E_a(s) = \left[\frac{K'_m \, K_f \, I_f \, K_b \, s}{(R_a + s\,L_a)} + J_m s^2 + s\,B_m \right] \theta_m(s)$$

$$\therefore \quad \boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{\dfrac{K_m}{s\,R_a\,B_m\,(1 + s\tau_m)(1 + s\,\tau_a)}}{1 + \dfrac{K_m \cdot sK_b}{s\,R_a\,B_m\,(1 + s\,\tau_m)(1 + s\,\tau_a)}} = \frac{G(s)}{1 + G(s)H(s)}}$$

where $\tau_m = J_m / B_m$, $\tau_a = \frac{L_a}{R_a}$ and $K_m = K'_m \, K_f$

$G(s) = \frac{K_m}{s\,R_a\,B_m\,(1 + s\,\tau_m)(1 + s\,\tau_a)}$ and $H(s) = sK_b$

Therefore T.F. can be represented in its block diagram form as in Fig. Q.20.2.

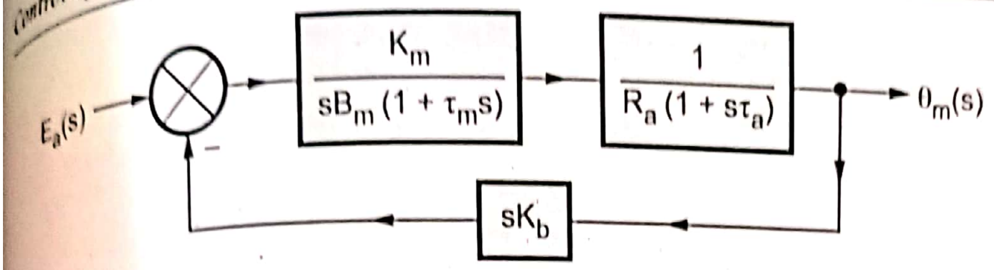

$E_a(s)$
$\frac{K_m}{sB_m(1+\tau_m s)}$
$\frac{1}{R_a(1+s\tau_a)}$
$\theta_m(s)$
−
sK_b

Fig. Q.20.2 Block diagram

Q.21 Derive the T.F. of field controlled d.c. servomotor.

☞ [SPPU : Dec.-13, 17, May-11, Marks 6]

Ans. : • The field controlled d.c. motor is shown in the Fig. Q.21.1.

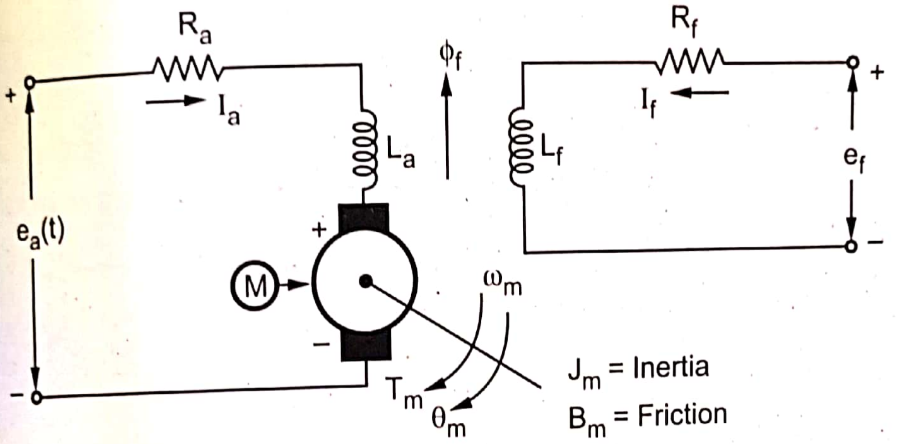

R_a
I_a
L_a
ϕ_f
R_f
I_f
L_f
+
e_f
−
+
$e_a(t)$
−
M
+
−
ω_m
T_m
θ_m
J_m = Inertia
B_m = Friction

Fig. Q.21.1

Assumptions :

1. Constant armature current is fed into the motor.
2. Flux produced is proportional to field current i.e. $\phi_f = K_f I_f$.
3. Torque is proportional to product of flux and armature current i.e. $T_m \propto \phi I_a$

$$\therefore \quad T_m = K' \phi I_a = K' K_f I_f I_a \quad \text{i.e.}$$

$$T_m = K_m K_f I_f \qquad \text{...(1)}$$

where $K_m = K' I_a$ = Constant

- Apply Kirchhoff's law to field circuit.

$$L_f \frac{di_f}{dt} + R_f I_f = e_f \qquad \text{... (2)}$$

Now shaft torque T_m is used for driving load against the inertia and frictional torque.

$$T_m = J_m \frac{d^2\theta_m}{dt^2} + B_m \frac{d\theta_m}{dt} \quad \text{...(3)}$$

$$\text{Inertia force} = J_m \frac{d^2\theta_m}{dt^2} \quad \text{similar to} \quad m\frac{d^2x}{dt^2}$$

$$\text{Frictional force} = B_m \frac{d\theta_m}{dt} \quad \text{similar to} \quad B\frac{dx}{dt}$$

- Finding Laplace transforms of equations (1), (2) and (3) we get,

$$T_m(s) = K_m K_f I_f(s) \quad \text{...(4)}$$

$$E_f(s) = (sL_f + R_f)\, I_f(s) \quad \text{...(5)}$$

$$T_m(s) = J_m s^2 \theta_m(s) + B_m s\, \theta_m(s) \quad \text{...(6)}$$

- Eliminate $I_f(s)$ from equations (4) and (5)

$$T_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)} \quad \text{...(7)}$$

- Eliminate $T_m(s)$ from equations (6) and (7),

$$(s^2 J_m + sB_m)\theta_m(s) = \frac{K_m K_f E_f(s)}{(sL_f + R_f)}$$

$$\text{Input} = E_f(s) \text{ and}$$

$$\text{Output} = \text{Rotational displacement } \theta_m(s)$$

$$\therefore \text{Transfer function} = \frac{\theta_m(s)}{E_f(s)}$$

$$\frac{\theta_m(s)}{E_f(s)} = \frac{K_m K_f}{(J_m s^2 + sB_m)(R_f + sL_f)}$$

$$= \frac{K_m K_f}{sR_f B_m [1 + s\tau_m][1 + s\tau_f]}$$

where $\tau_m = \frac{J_m}{B_m}$ = Motor time constant, $\tau_f = \frac{L_f}{R_f}$ = Field
constant

$$\boxed{\text{T.F.} = \frac{\theta_m(s)}{E_f(s)} = \frac{K_f}{R_f[1 + s\tau_f]} \cdot \frac{K_m}{B_m(1 + s\tau_m)} \cdot \frac{1}{s}}$$

7.14 : Synchros

Q.22 Explain the working principle of synchros and how it works as an error detector.

☞ **[SPPU : April-17 (Insem), Dec.-14, 18, May-15, Marks 8]**

Ans. : • Synchro is basically a rotary device, an electromagnetic transducer which operates on same principle as that of transformer. It converts angular position of shaft into an electric signal.

- The basic synchro unit has two part, stator and rotor.
- The stator which is stationary part is made up of laminated steel. This part is slotted to accommodate a balanced three phase winding.
- The rotor which is rotating part is a salient pole, dumb-bell shaped magnet with a single winding.
- The schematic diagram of synchro transmitter is shown in the Fig. Q.22.1.

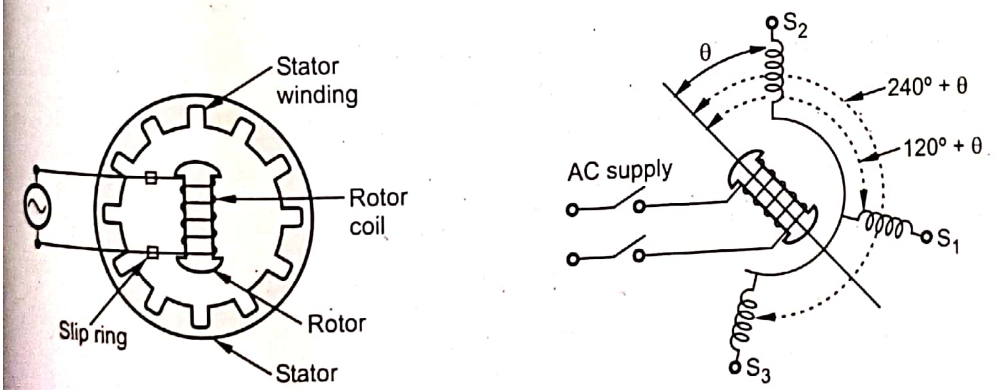


Fig. Q.22.1 Synchro transmitter

- Let a.c. voltage applied to rotor is, $e_r(t) = E_r \sin \omega_0 t$
- When $\theta = 0°$ with reference to schematic diagram, voltage induced in s_2 winding will be maximum and position is called as electric zero.

- The applied voltage causes a flow of magnetising current in rotor coil which produces sinusoidally varying flux. This flux gets distributed sinusoidally in air gap.

- Because of transformer action voltages get induced in all windings which is proportional to cosine of angle between rotor and stator coil axes.

- Consider now that the rotor of synchro transmitter is at an angle 'θ' with reference to electric zero then voltages in each stator coil w.r.t. neutral are,

$$E_{S_{1n}} = K E_r \cos(\theta - 240) \sin \omega_0 t$$

$$E_{S_{2n}} = K E_r \cos\theta \sin \omega_0 t$$

$$E_{S_{3n}} = K E_r \cos(\theta - 120) \sin \omega_0 t$$

- Magnitudes of terminal stator voltages are,

$$E_{s_1 s_2} = E_{s_{1n}} - E_{s_{2n}} = \sqrt{3} K E_r \sin(\theta + 240^\circ) \sin \omega_0 t$$

$$E_{s_2 s_3} = E_{s_{2n}} - E_{s_{3n}} = \sqrt{3} K E_r \sin(\theta + 120^\circ) \sin \omega_0 t$$

$$E_{s_3 s_1} = E_{s_{3n}} - E_{s_{1n}} = \sqrt{3} K E_r \sin\theta \sin \omega_0 t$$

- Thus it is seen that the input to the synchro transmitter is the angular position of its rotor shaft and output is a set of three single phase voltages the magnitudes of these voltages are functions of the shaft position.

Synchros as an error detector :

- An error detector involves both synchros synchro transmitter as well as synchro control transformer.

- Synchro control transformer is an electro mechanical device which produces a single phase voltage whose magnitude is proportional to the sine of the angle of rotation of its rotor with respect to stator magnetic field. The symbol CT is used to indicate synchro control transformer.

- The Fig. Q.22.2 shows schematic diagram of synchro error detector in which second component is synchro control ransformer. The output of synchro transmitter is given to nchro control transformer.

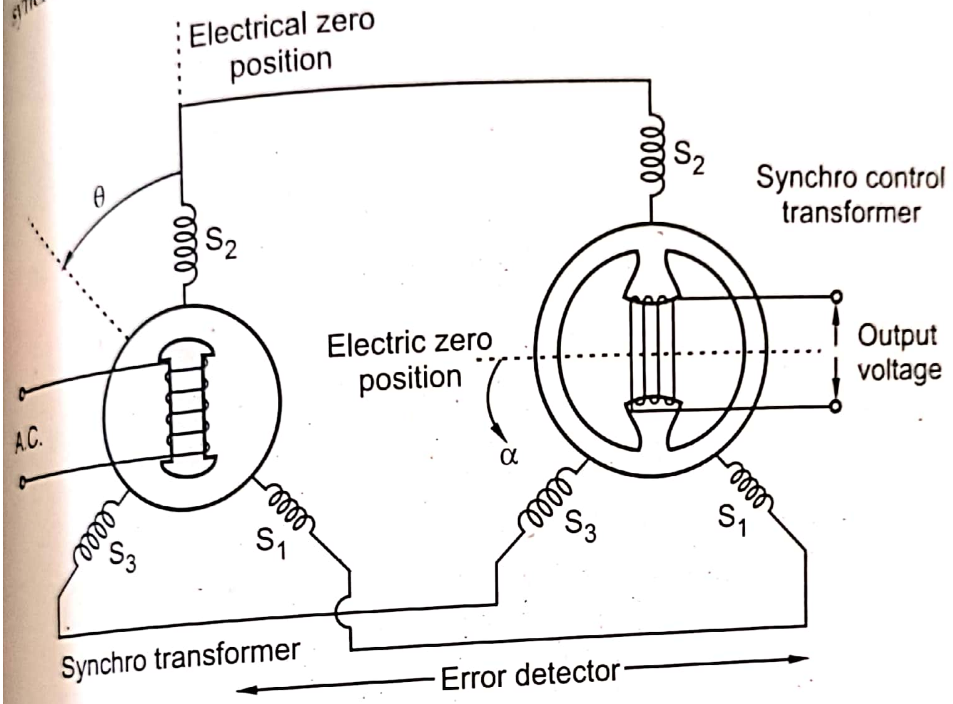

Electrical zero position
θ
S2
S2
Synchro control transformer
Electric zero position
Output voltage
A.C.
α
S3
S1
S3
S1
Synchro transformer
Error detector

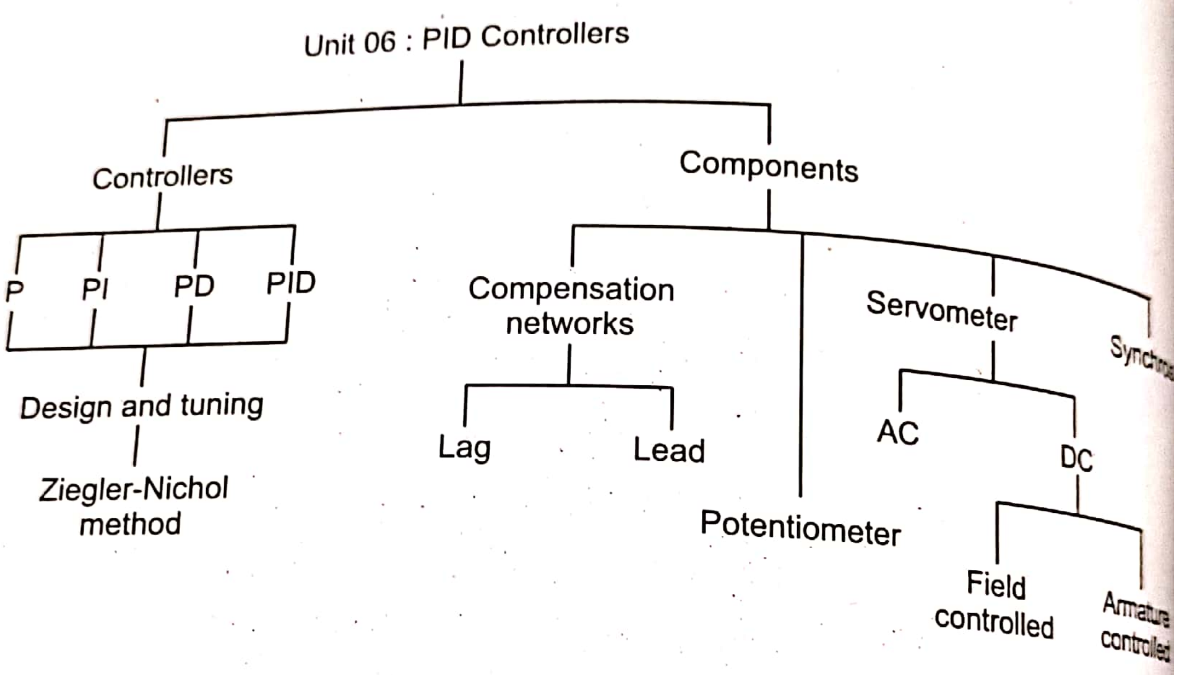
Fig. Q.22.2 Synchro as error detector

- When the rotor positions of the two synchros are in perfect alignment, the voltage generated across the terminals of the rotor of control transformer is zero.
- Let the transmitter rotate through an angle 'θ' in the direction indicated and let control transformer rotor rotates in the same direction through an angle 'α'.
- So net angular separation of the two rotors $\phi = (90^\circ - \theta + \alpha)$
- Now in general voltage across rotor terminals of control transformer is,

$e(t) = K' v_r \cos\phi \sin\omega_0 t = K' v_r \sin(\theta - \alpha) \sin\omega_0 t$ ...substituting ϕ

So when two rotor shafts are not in alignment, the rotor voltage of 'CT' is approximately a sine function of the difference between the two shaft angles.

Memory Map


Unit 06 : PID Controllers
Controllers
Components
P
PI
PD
PID
Design and tuning
Ziegler-Nichol method
Compensation networks
Lag
Lead
Potentiometer
Servometer
AC
DC
Field controlled
Armature controlled
Synchro

END...

APRIL - 2016 [IN SEM] [15] Solved Paper

2012 Course

Time : 1 Hour] [Maximum Marks : 30

Instructions to the candidates :

1) *Attempt Q. 1 or Q. 2, Q. 3 or Q. 4, Q. 5 or Q. 6.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Your answers will be valued as a whole.*

Q.1 a) *Classify open loop and close system with their advantages and limitations.* **(Refer Q.5 of chapter 1)** **[4]**

b) *Find transfer function for following Fig. 1.*

(Refer Q.25 of chapter 1) **[6]**

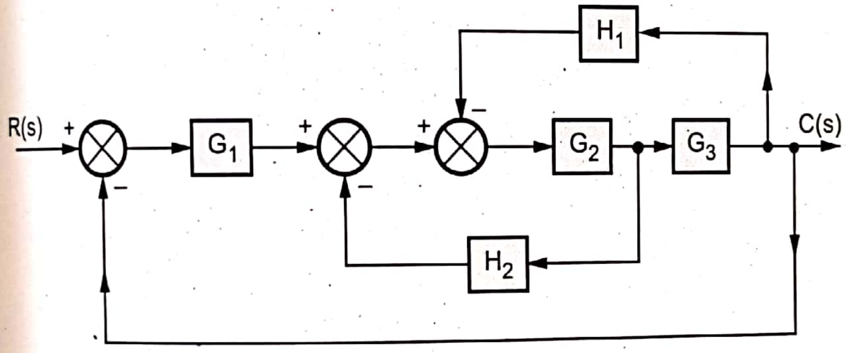


Fig. 1

OR

Q.2 a) *State and explain Masons gain formula.*

(Refer Q.33 of chapter 1) **[4]**

b) *Find transfer function Mason's gain formula following system.*

(Refer Q.34 of chapter 1) **[6]**

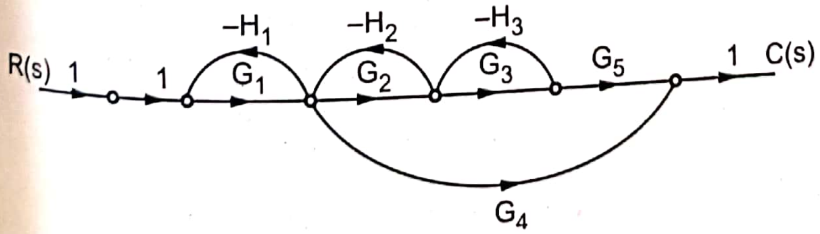


Fig. 2

Q.3 a) *Derive transfer function for separately excited DC shunt motor.* **(Refer Q.20 of chapter 7)** [7]

b) *Explain force voltage analogy.* **(Refer Q.15 of chapter 1)** [3]

OR

Q.4 a) *Derive transfer function for following electrical system.* **(Refer Q.10 of chapter 1)** [6]

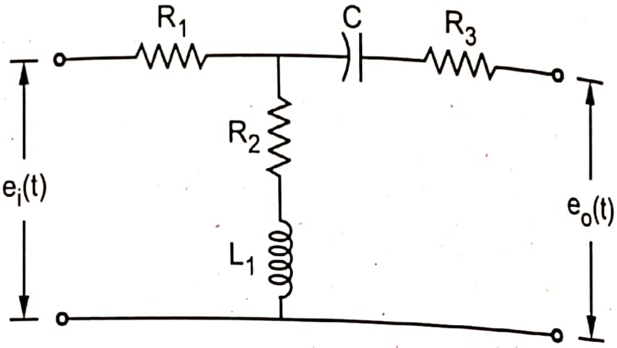


Fig. 3

b) *Explain gear train and how they are modelled.* **(Not in new syllabus)** [4]

Q.5 a) *Draw time response of standard second order system and explain time response specifications.* **(Refer Q.12 of chapter 2)** [5]

b) *For a unity feedback systems having open loop transfer function as*

$$G(s) = \frac{24\,(s+2)}{s\,(s^2+7s+12)}$$

Determine : i) Type of system ii) Error constant K_p, K_v *and* K_a *iii) Steady state error for step input.* **(Refer Q.6 of chapter 2)** [5]

OR

Q.6 a) *A unity feedback system is characterized by an open loop transfer function* $G(s) = \frac{K}{s(s+10)}$. *Determine the gain K so that the system will have damping ratio 0.7. From this value of K determine settling time, peak overshoot and time to peak overshoot.* **(Refer Q.18 of chapter 2)** [6]

b) *What are different standard test signals ? Draw them and find their laplace transform.* **(Refer Q.1 of chapter 2)** [4]

MAY - 2016 [END SEM] [4958] - 1069

Solved Paper

2012 Course

ime : $2\frac{1}{2}$ Hours [Maximum Marks : 70

Q.1 a) *Compare open loop system with close loop system. Give one xample of each.* **(Refer Q.5 of chapter 1)** **[6]**

b) *Derive transfer function of lead network.*

Refer Q.17 of chapter 7) **[7]**

c) *Explain different static error coefficients and steady state error n each case.*

(Refer Q.3 of chapter 2) **[7]**

OR

Q.2 a) *Find transfer function of* **(Refer Q.22 of chapter 1)** **[7]**

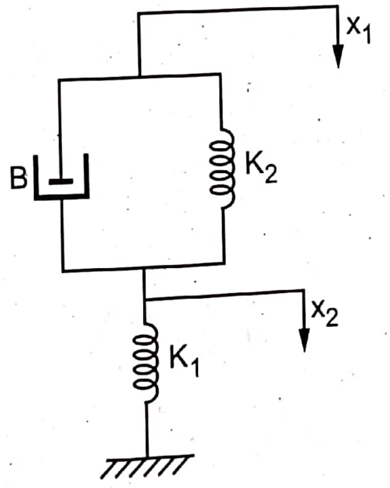


Fig. 1

b) *Derive transfer function of DC servomotor.* **[6]**

(Refer Q.20 of chapter 7)

c) *Find damping ratio, settling time and peak overshoot for system with close loop transfer function given by* $T(S) = \dfrac{10}{s^2+7s+20}$ **[7]**

(Refer Q.16 of chapter 2)

Q.3 **a)** *Explain Routh Hurwitz stability criterion.*

(Refer Q.4 of chapter 3) [6]

b) *Draw root locus for following system. Also find range of values of K for which system is stable.* $G(S) = \dfrac{K}{s(s+2)(s^2+s+1)}$

(Refer Q.21 of chapter 3) [10]

OR

Q.4 **a)** *Using Routh Hurwitz criterion determine whether the given close loop system is unstable. If unstable how many poles are on right half s plane or imaginary axis.* **(Refer Q.9 of chapter 3)**

$$G(S) = \frac{10}{s^5+7s^4+6s^3+42s^2+8s+56}$$

[8]

b) *Explain rules for construction of root locus.*

(Refer Q.14 of chapter 3) [8]

Q.5 **a)** *Draw Bode plot for following system* $G(S) = \dfrac{1000}{s(1+0.1s)(1+0.001s)}$ *Find gain margin and phase margin comment on stability.* **(Refer Q.10 of chapter 5)** [12]

b) *Define Gain margin and phase margin. Explain how it is to be found using Bode plot.* **(Refer Q.4 and Q.5 of chapter 5)** [6]

OR

Q.6 **a)** *Draw polar plot for* $G(S) = \dfrac{10}{s(s+1)(s+4)}$. [9]

b) *Explain Nyquist stability criterion. Explain procedure for drawing nyquist plot and how to determine stability.*

(Refer Q.8 and Q.10 of chapter 6) [9]

Q.7 **a)** *Explain P and PI controllers. Explain their effect on damping ratio and steady state error.* **(Refer Q.1 of chapter 7)** [8]

b) *Design a PID controller for system with unity feedback and*

$G(S) = \dfrac{K}{(s+3)(s^2+s+1)}$. **(Refer Q.14 of chapter 7)** [8]

OR

Q.8 **a)** *Explain Ziegler Nichols method of tuning PID controller.* **(Refer Q.13 of chapter 7)** **[8]**

b) *Explain procedure for designing PID controller using root locus.* **(Refer Q.12 of chapter 7)** **[8]**

DECEMBER - 2016 [END SEM] 5058 - 349 Solved Paper

Course 2012

Time : $2\frac{1}{2}$ Hours] **[Maximum Marks : 70**

Instructions to the candidates :

1) *Answer all questions*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*

Q.1 **a)** *Distinguish between.* **[6]**

i) Open loop and closed loop system. **(Refer Q.5 of chapter 1)**

ii) Feedback and feedforward system. **(Refer Q.8 of chapter 1)**

b) *Write a short note on tachogenerator.* **(Not in New Syllabus)** **[6]**

c) *A unity feedback system characterised by the open loop transfer function.* **[8]**

$$G(s) = \frac{1}{s(0.5\,s+1)(0.2\,s+1)}$$

Determine the steady state errors for unit step, unit ramp and unit acceleration inputs. **(Refer Q.8 of chapter 1)**

OR

Q.2 **a)** *Draw mechanical equivalent network of a given system and also draw the electrical analogous circuit using F-V analogy.* **[8]**
(Refer Q.18 of chapter 1)

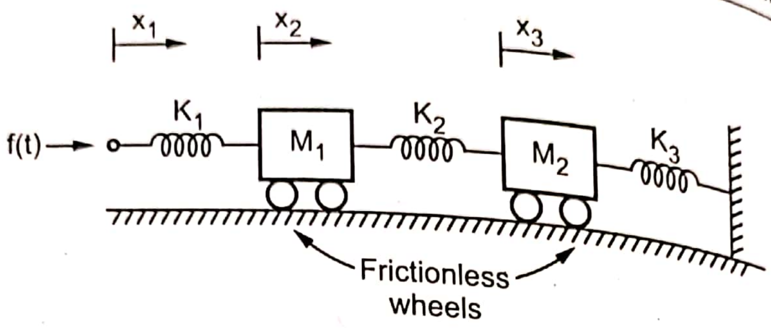


Fig. 1

b) *State and explain Mason's Gain Formula.*
(Refer Q.33 of chapter 1)

c) *Define and explain time domain specification.* [5]
(Refer Q.12 of chapter 2)

Q.3 a) *State and explain Routh Hurwitz stability criterion.* [7]
(Refer Q.4 of chapter 3)

b) *A unity feedback control system has a open loop transfer function.* [6]

$$G(s) = \frac{K}{s(s^2+4s+13)}$$ [10]

Sketch the root locus plot of the system by determining the following :

i) Centroid, number and angle of asymptotes.

ii) Angle of departure of root loci from the poles.

iii) Breakaway point if any

iv) The value of K and the frequency at which the root loci cross the jω-axis. **(Refer Q.19 of chapter 3)**

OR

Q.4 a) *Sketch the root locus for the open-loop transfer function of a unity feedback control system given below and determine.*

i) The value of K for $\xi = 0.5$ *ii) the value of K for marginal stability* [9]

$$G(s) = \frac{K}{s(s+1)(s+3)}$$ **(Refer Q.17 of chapter 3)**

b) *The open-loop transfer function of a unity feedback system is given by :* $G(s) = \dfrac{K}{(s+2)(s+4)(s^2+6s+25)}$

By applying the Routh criterion discuss the stability of the closed loop system as a function of K. Determine the value of K which will cause sustained oscillation in the closed loop system. What are the corresponding oscillation frequencies ? **(Refer Q.12 of chapter 3)** **[7]**

Q.5 a) *Explain the terms gain margin, phase margin, Gain cross over frequency.* **(Refer Q.4 of chapter 5)** **[8]**

b) *Sketch the Bode plot of the transfer function and determine PM and GM from the plot.*

$$G(s)H(s) = \frac{30}{s(1+0.5s)(1+0.08s)}$$ **(Refer Q.6 of chapter 5)** **[10]**

OR

Q.6 a) *Explain Nyquist stability criteria.* **[8]**

(Refer Q.8 of chapter 6)

b) *A control system with closed loop transfer function.*

$$G(s)H(s) = \frac{K(s-1)}{s(s+1)}$$

Plot Nyquist plot and comment on stability. **[10]**

(Refer Q.13 of chapter 6)

Q.7 a) *Draw block diagram of PID controller and discuss the effect of increasing* K_p, K_s, K_h *on rise time overshoot and stability.* **[8]**

(Refer Q.6 of chapter 7)

b) *The system given below is so design of have damping ratio 0.707. Determine the required values of* K_p *for the given damping ratio.* **[8]**

(Refer Q.8 of chapter 7)

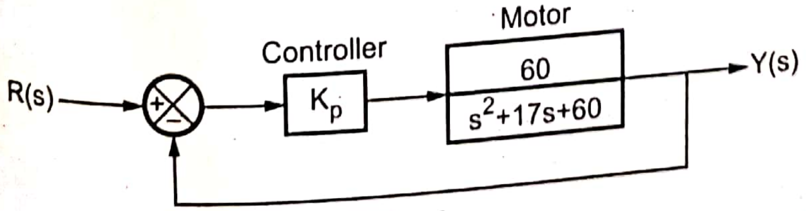


Fig. 2

OR

Q.8 **a)** *Explain Ziegler Nichols method of tuning P.* **(Refer Q.13 of chapter 7)**

b) *An open loop test of a temperature control system yields the reaction curve shown below. The system open loop transfer function is given by*

$$G(s) = \frac{2}{(20s+1)(50s+1)}$$

Used Ziegler Nichols method to determine K_p, K_i, K_d ? For a quarter step response PID control system. **(Refer Q.15 of chapter 7)**

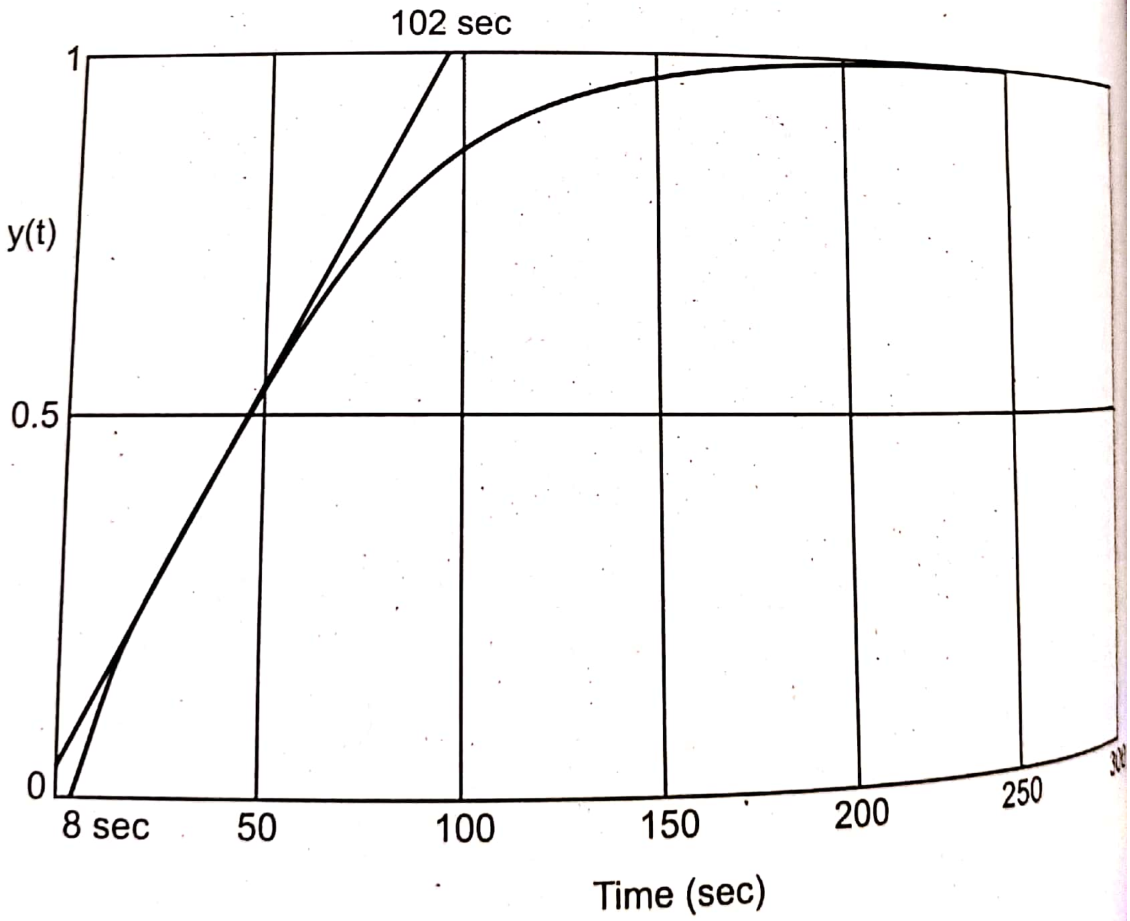


Fig. 3

APRIL - 2017 [IN SEM] [15] Solved Paper

Course 2012

Time : 1 Hour] [Maximum Marks : 30

Instructions to the candidates :

1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*

Q.1 a) *Compare :* **[4]**

i) Feedback and Feedforward control system.

(Refer Q.8 of chapter 1)

ii) Open loop and Closed loop control system.

(Refer Q.5 of chapter 1)

b) *Obtain the overall transfer for a system represented by a block diagram.* (Refer Q.27 of chapter 1) **[6]**

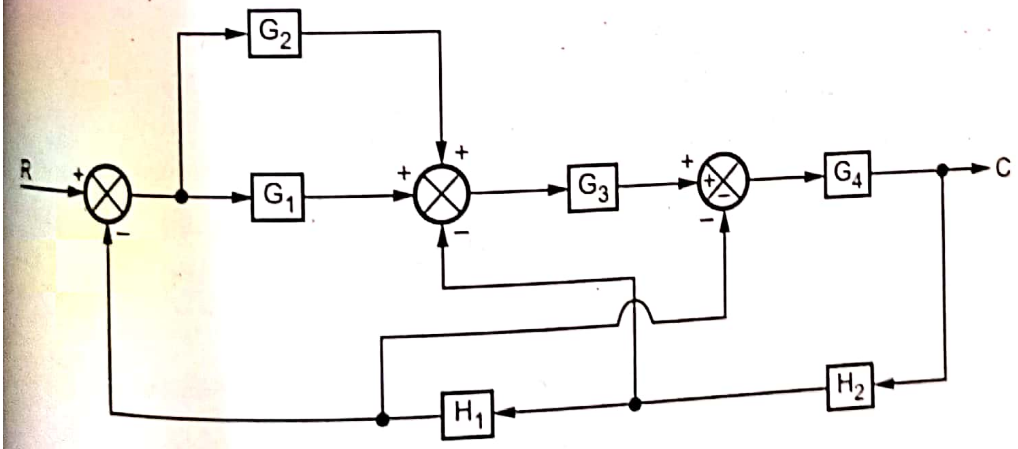


Fig. 1

OR

Q.2 a) *Define :* **[4]**

i) Transfer function (Refer Q.9 of chapter 1)

ii) Pole and Zero of a system (Refer Q.13 of chapter 1)

b) *Obtain a mathematical model for the mechanical system shown. Write DE, using F-V analogy. Draw equivalent mechanical circuit.* **(Refer Q.19 of chapter 1)** **[6]**

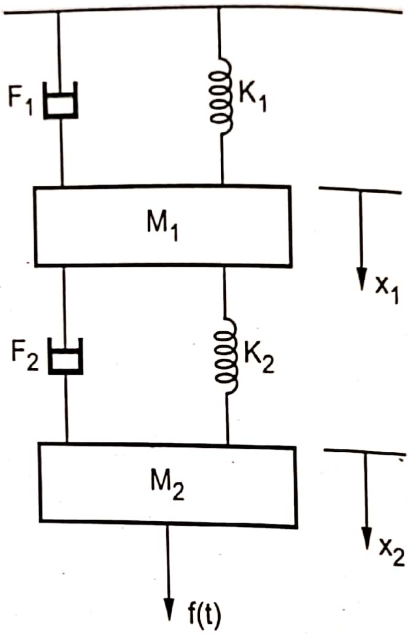


Fig. 2

Q.3 a) *What is lead network ? Obtain the TF of lead network.* **(Refer Q.17 of chapter 7)** **[4]**

b) *Obtain the block diagram representation of AC servomotor.* **(Refer Q.19 of chapter 7)** **[6]**

OR

Q.4 a) *Write a short note on synchro transmitter receiver.* **(Refer Q.22 of chapter 7)** **[6]**

b) *Explain AC tachogenerator.* **(Not in new syllabus)** **[4]**

Q.5 a) *Write the expression for closed loop TF for first order system and the effect of time constant on system performance.* **(Refer Q.10 of chapter 2)** **[4]**

b) *A unity feedback system is characterised by an open loop.*

$$TF = K/s\,(s + 10)$$

Determine the gain K so that the system will have a damping ratio of 0.5 for this value of K determine settling time, Peak overshoot and time to peak overshoot for a unit step input. **(Refer Q.17 of chapter 2)** **[6]**

a) The block diagram for a unity feedback control system is shown **[6]**

ig. 3.

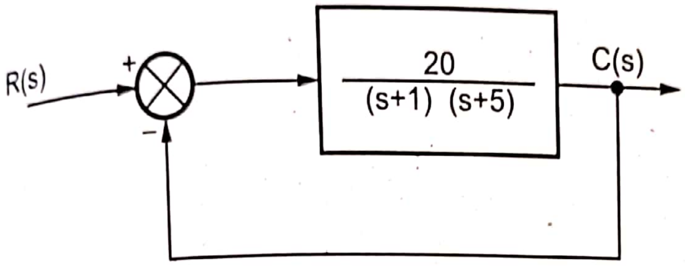


Fig. 3

ermine the characteristic equation of the system, ω_n, ξ ω_d, t_p, M_p, the e at which the first overshoot occurs, the time period of oscillations d the number of cycles completed before reaching the steady state.

efer Q.19 of chapter 2)

b) *Define static error constant and write the expression for them.*

efer Q.3 of chapter 2) **[4]**

MAY - 2017 END SEM [5153]-567 Solved Paper

Course 2012

ime : 2 $\frac{1}{2}$ Hours] **[Maximum Marks : 70**

.1 a) *Derive transfer function using block diagram reduction.* **[6]**

Refer Q.26 of chapter 1)

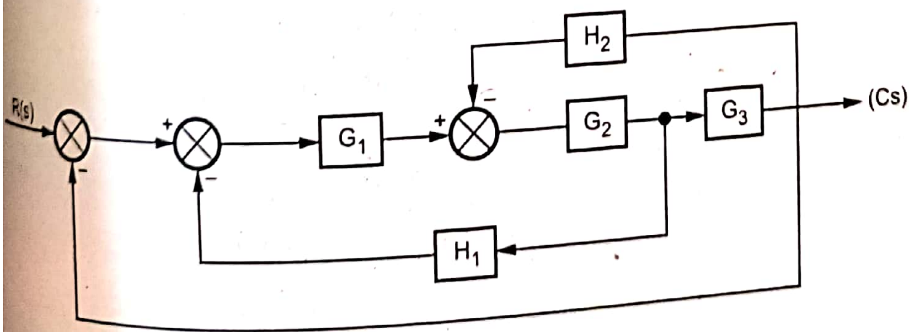


Fig. 1

b) *Derive transfer function of DC servomotor (armature control).* **(Refer Q.20 of chapter 7)** [7]

c) *Derive time response of unit step input to first order system. Sketch the response.* **(Refer Q.10 of chapter 2)** [7]

OR

Q.2 a) *Explain F-V analogy and F-I analogy.* **(Refer Q.15 and Q.16 of chapter 1)** [6]

b) *Determine transfer function of following electrical network.* **(Refer Q.11 of chapter 1)** [7]

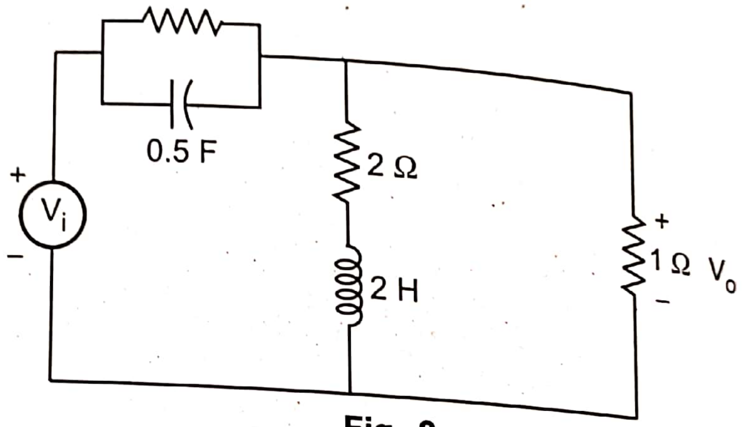


Fig. 2

c) *What is type and order of system. Explain effect of type of system on steady state error.* **(Refer Q.4, Q.5 of chapter 2)** [7]

Q.3 a) *Define stability and give necessary conditions for stability.* **(Refer Q.3 of chapter 3)** [4]

b) *Explain for unity feedback system with open loop transfer function given as* $G(s) = \dfrac{K}{s(s+1)(s^2+4s+13)}$. *Draw root locus when K is varied from 0 to ∞. Also find range of values of K for which system is stable.* **(Refer Q.20 of chapter 3)** [12]

OR

Q.4 a) *Using Routh Hurwitz criterion for the unity feedback control system with open loop transfer function* $G(s) = \dfrac{K(s+13)}{s(s+3)(s+7)}$

i) Find range of values of K for the system to be stable

ii) *Find the value of K for marginally stable system and sponding close loop poles and frequency of sustained oscillations.*
fer Q.11 of chapter 3) **[8]**

b) *Explain any four rules for root locus.*
fer Q.14 of chapter 3) **[8]**

5 a) *Draw bode plot for following system* $G(s) = \frac{20(s+2)}{s(s+10)}$ *Find gain gin and phase margin comment on stability.*
fer Q.11 of chapter 5) **[12]**

b) *Explain different frequency domain specifications.*
fer Q.4 of chapter 5) **[6]**

OR

6 a) *Explain how gain margin and phase margin are determined in de plot and stability from that.* **(Refer Q.5 of chapter 5)** **[6]**

b) *Explain Nyquist stability criterion. Sketch Nyquist plot for the stem with open loop transfer function given by* $GH(s) = \frac{20}{(s+2)(s+3)}$ *mment on stability of system.* **(Refer Q.14 of chapter 6)** **[12]**

7 a) *Explain P, PI, PID controller and their features.*
Refer Q.1, Q.4, Q.6, Q.7 of chapter 7) **[8]**

b) *A unity feedback system has the plant transfer function* $G(s) = \frac{C(s)}{M(s)} = \frac{10}{s\,(s+2)}$. *A proportional plus derivative control is employed to ntrol the dynamics of the system. Determine*
) *The damping factor and undamped natural frequency when* $K_d = 0$
) *The value of* K_d *such that damping factor is 0.6.* **[8]**
Refer Q.10 of chapter 7)

OR

Q.8 a) *Explain Ziegler Nichols method of tuning PID controller.* **[8]**
(Refer Q.13 of chapter 7)

b) *Using Ziegler Nichols method design a PID controller for ystem with open loop transfer function* $G(s) = \frac{15}{s(s+1)(s+3)}$. *Write close loop transfer function of plant including PID controller.* **[8]**
(Refer Q.16 of chapter 7)

DECEMBER - 2017 [END SEM] [5253] - 167

Solved Paper

Course 2012

Time : 2½ Hours] [Maximum Marks : 70

Q.1 a) *Using block diagram reduction technique find the transfer function of* **(Refer Q.28 of chapter 1)** **[6]**

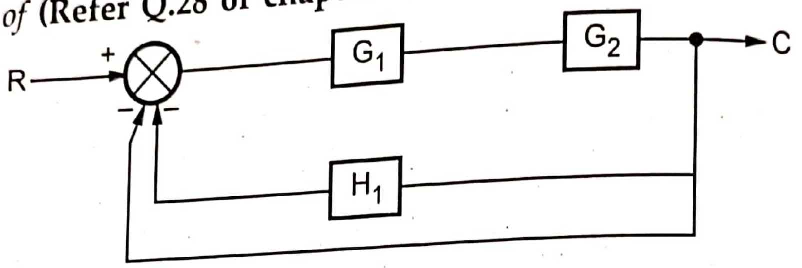


Fig. 1

b) *Compare open loop and close loop control system.* **(Refer Q.5 of chapter 1)** **[4]**

OR

Q.2 a) *Find transfer function of given mechanical system between* x_2 *and f(t).* **(Refer Q.20 of chapter 1)** **[6]**

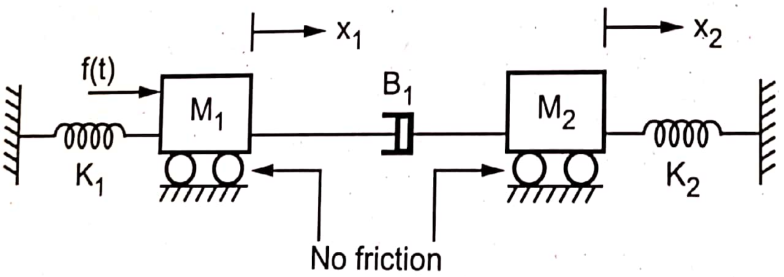


Fig. 2

b) *Explain force voltage analogy.* **(Refer Q.15 of chapter 1)** **[4]**

Q.3 a) *Draw time response of second order system for*

i) $\xi = 1$ *ii)* $\xi < 1$ *iii)* $\xi > 1$ *iv)* $\xi = 0$

Comment on response of system from that. **(Refer Q.11 of chapter 2)** **[6]**

b) *Explain lag and lead netwrok with their transfer function.* **(Refer Q.17 of chapter 7)** **[4]**

OR

Q.4 a) *Derive transfer function of DC servomotor.* **(Refer Q.21 of chapter 7)** **[6]**

b) *For unity feedback system* $G(s) = \dfrac{100}{s^2 + 15s + 100}$ *find* **[4]**

i) Peak overshoot ii) Setling time iii) Peak time
(Refer Q.14 of chapter 2)

Q.5 a) *Sketch the root locus of unity feedback system whose open loop transfer function is* $G(s) = \dfrac{K}{s(s+1)(s^2+4s+13)}$. *Find marginal value of K for the system to be stable.* **(Refer Q.20 of chapter 3)** **[10]**

b) *Explain Routh Hurwitz criterion for determining stability of system.* **(Refer Q.4 of chapter 3)** **[6]**

OR

Q.6 a) *Using Routh's Hurwitz criterion for stability analysis determine stability of following system* $P(s) = s^5 + 3s^4 + 5s^3 + 4s^2 + s + 3$. *From that determine how many roots are there in left half s plane, right half s plane, and on imaginary axis.* **(Refer Q.5 of chapter 3)** **[8]**

b) *Explain any 4 rules for construction of root locus.*
(Refer Q.14 of chapter 3) **[8]**

Q.7 a) *Explain polar plot. Draw polar plot for system with open loop transfer function.* **[8]**

$G(s) = \dfrac{50}{s(s+3)(s+6)}$ **(Refer Q.1 and Q.4 of chapter 6)**

b) *Explain the nature of Bode plots for* **[10]**
i) Poles at origin ii) Simple pole and iii) Simple zero
(Refer Q.3 of chapter 5)

OR

Q.8 a) *Find stability of the following system by sketching the Bode plot.* **(Refer Q.7 of chapter 5)** **[12]**

$G(s) = \dfrac{200}{s(s+10)(s+2)}$

b) *State and explain Nyquist stability criterion.*
(Refer Q.8 of chapter 6) **[6]**

Q.9 a) *Explain PID controller with its characteristics and its effect on system performance.* **(Refer Q.4, Q.6 of chapter 7)** **[8]**

b) *Explain P, PI, PID controller.* **(Refer Q.1 of chapter 1)** **[8]**

OR

Q.10 a) *A second order system has an open loop transfer function* $G(s) = \frac{K}{s(0.2s+1)}$ *where K is proportional gain. Find the value of K so that steady state error will be < 0.5 when input is* $\frac{\pi}{6}t$.

(Refer Q.9 of chapter 7) **[8]**

b) *Explain tuning of PID controllers using Ziegler-Nichols method.* **(Refer Q.13 of chapter 7)** **[8]**

APRIL - 2018 [IN SEM] 116 Solved Paper

2015 Course

Time : 1 Hour] **[Maximum Marks : 30**

Instructions to the candidates :

1) *Answer Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6.*
2) *Neat diagrams must be drawn wherever necessary.*
3) *Figures to the right indicate full marks.*
4) *Assume suitable data, if necessary.*

Q.1 a) *Compare : Feedback and Feed forward control system.* **(Refer Q.8 of chapter 1)** **[4]**

b) *Determine equilibrium equations for following system using force - current analogy.* **(Refer Q.21 of chapter 1)** **[6]**

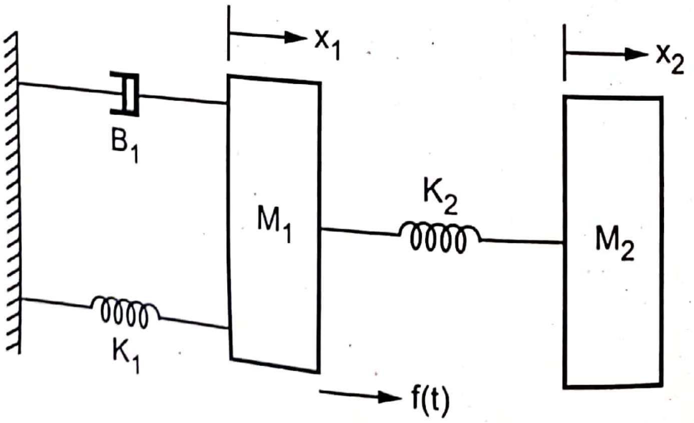


Fig. 1

OR

Q.2 **a)** *State and explain Mason's gain formula for Signal flow graph.* **(Refer Q.33 of chapter 1)** **[4]**

b) *Obtain transfer function of the system.* **(Refer Q.29 of chapter 1)**

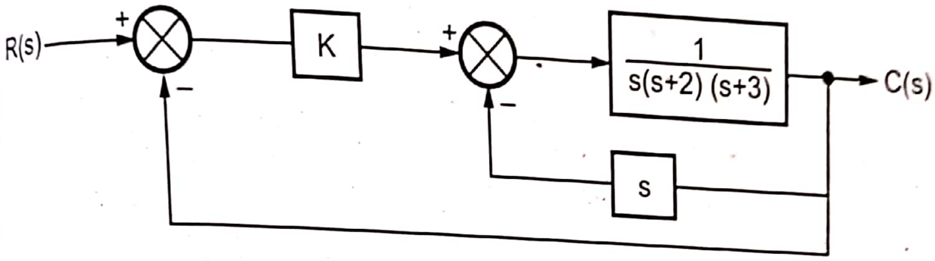


Fig. 2

[6]

Q.3 **a)** *Derive an expression of steady state error for a simple closed loop system.* **(Refer Q.2 of chapter 2)** **[4]**

b) *A unity feedback system has* $G(s) = \dfrac{K}{s(s+1)(1+0.4\,s)}$. **[6]**

i) For ramp input $r(t) = 4t$ *and* $K = 2$*, find steady state error.*

ii) If desired steady state error to be 0.2, Find corresponding value of K. **(Refer Q.9 of chapter 2)**

OR

Q.4 **a)** *Explain effect of* ξ *(damping factor) on locations of closed loop poles.* **(Refer Q.11 of chapter 2)** **[4]**

b) *For system having* $\dfrac{C(s)}{R(s)} = \dfrac{20}{s^2+7s+25}$*, determine all time domain specifications.* **(Refer Q.15 of chapter 2)** **[6]**

Q.5 **a)** *How two special cases of Routh's criterion can be solved ?* **(Refer Q.6 of chapter 3)** **[4]**

b) *Determine the range of K for stability of a system having characteristic equation* $s^3 + 3Ks^2 + (K+2)s + 4 = 0$. *Find the value of K so that the system is marginally stable and find the frequency of sustained oscillations.* **(Refer Q.10 of chapter 3)** **[6]**

OR

Q.6 a) *Explain the terms : Centroid and angle of asymptotes of root locus.* **(Refer Q.14 of chapter 3)** [4]

b) *Determine break away point and angle of departure for root locus of a system with open loop transfer function.*

$$G(s)H(s) = \frac{K(s+1)}{s^2+4s+13}$$ **(Refer Q.15 of chapter 3)** [4]

MAY - 2018 [END SEM] 5353 - 567 Solved Paper

Course 2015

Time : $2\frac{1}{2}$ Hours] [Maximum Marks : 70

Q.1 a) *Find the transfer function $V_o(s)/V_i(s)$ of the circuit shown in Fig. 1, if L = 1 H and C = 1F.* **(Refer Q.12 of chapter 1)** [7]

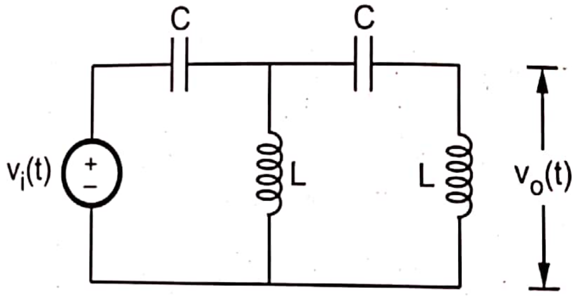


Fig. 1

b) *Draw standard test signals, write expressions of standard tes signals in time domain and write their Laplace transform. For wha purpose these signals are used ?* **(Refer Q.1 of chapter 2)** [6

c) *By means of Routh criterion, determine the stability of th system represented by the following characteristic equation* $s^6+2s^5+8s^4+12s^3+20s^2+16s+16$ **(Refer Q.8 of chapter 3)** [7

OR

Q.2 a) *Using Mason's rule, find the transfer function $T(s) = C(s)/R($ for the system represented in the Fig.2.* **(Refer Q.37 of chapter 1)** [7

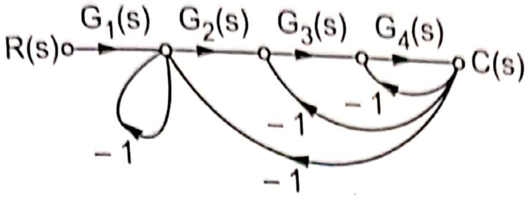


Fig. 2

b) *Derive expression for rise time and peak overshoot for a second order underdamped system for unit step input.*

(Refer Q.13 of chapter 2) **[6]**

c) *Sketch the root locus of the unity feedback system having.* $G(s)=\dfrac{K}{s(s+2)(s+4)}$ *where K is varied from 0 to ∞. Hence obtain the value of K for which the system is unstable.* **(Refer Q.18 of chapter 3)**

[7]

Q.3 **a)** *Draw polar plot of the given system.* **[10]**

$$G(s)H(s) = \frac{60}{(s+1)(s+2)(s+5)}$$

Find the frequency and the magnitude of G(s)H(s)

i) at which the plot intersects y axis

ii) at which the plot intersects x axis. **(Refer Q.5 of chapter 6)**

b) *Explain with sketch the contour which encloses entire right half of s-plane and state Nyquist stability criteria.*

(Refer Q.8 and Q.9 of chapter 6) **[6]**

OR

Q.4 **a)** *Consider the following open loop system* **[8]**

$$G(s)H(s) = \frac{s+2}{(s+1)(s-1)}$$

Draw Nyquist plot and comment on the stability of closed loop system. **(Refer Q.15 of chapter 6)**

b) *Draw a typical frequency response magnitude characteristics and explain frequency response specifications. And also write relation between time and frequency response for second order system.*

(Refer Q.4 of chapter 5 and Q.4 of chapter 4) **[8]**

Q.5 **a)** *The open loop unity feedback system is* **[12]**

$$G(s) = \frac{2000}{s(s+1)(s+100)}$$

Draw Bode plot, determine the followig and comment on the stability.

i) Gain cross over frequency *ii) Phase cross over frequency*

iii) Gain margin *iv) Phase margin.* **(Refer Q.9 of chapter 5)**

b) *Explain how to draw the Bode magnitude and phase angle plots of a quadratic pole.* **(Refer Q.3 of chapter 5)** **[6]**

OR

Q.6 **a)** *The open loop unity feedback system is* $G(s) = \frac{K}{s(s+1)(s+10)}$

Draw Bode plot and determine : i) The value of K for gain margin of 7 dB. ii) The value of K for phase margin of 40 °C.

(Refer Q.12 of chapter 5) **[12]**

b) *Explain the concept of gain margin and phase margin. Explain how these values help in studying relative stability. Mark phase and gain margin on Bode plots for an unstable system.*

(Refer Q.4 and Q.5 of chapter 5) **[6]**

Q.7 **a)** *Design PID controller for unity feedback system given below using Zeigler-Nichols tuning method.* **[8]**

$$G(s) = \frac{15}{s(s+1)(s+3)}$$ **(Refer Q.16 of chapter 7)**

b) *Derive the transfer function of armature control D.C. servo motor.* **(Refer Q.20 of chapter 7)** **[8]**

OR

Q.8 **a)** *Draw block diagram of PID controller and write the expression for output signal from PID controller in time domain. Explain effect of proportional, integral and derivative control on rise time, peak overshoot, setting time and steady state error.*

(Refer Q.2,Q.3 and Q.7 of chapter 7) **[8]**

b) *Draw the circuit diagram of lead compensator network and derive transfer function also plot pole zero locations in s-plane.*

(Refer Q.17 of chapter 7) **[8]**

ECEMBER - 2018 [END SEM] 5460 - 566

Solved Paper

Course 2015

me : $2\frac{1}{2}$ Hours] [Maximum Marks : 70

1 a) *Derive an expression for force - voltage analogy in translational d rotational systems using simple R - L - C series circuit.*

efer Q.15 of chapter 1) [8]

b) *Draw signal flow graph and obtain the tranfer function using ason's Gain Formula for the block diagram shown below :* [8]

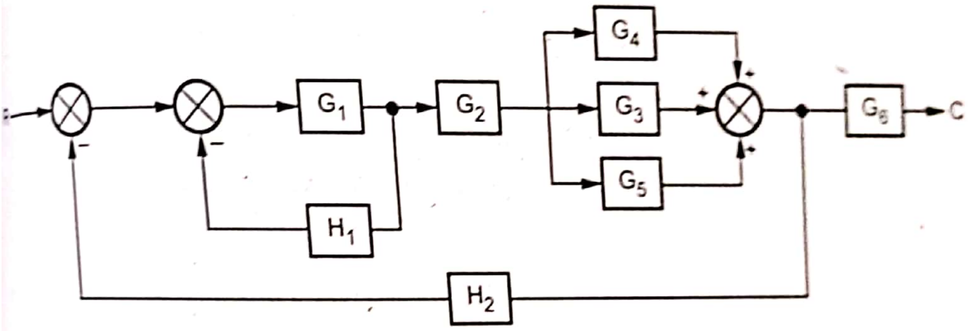


Fig. 1

Ans. : The signal flow graph is shown in the Fig. 1(a)

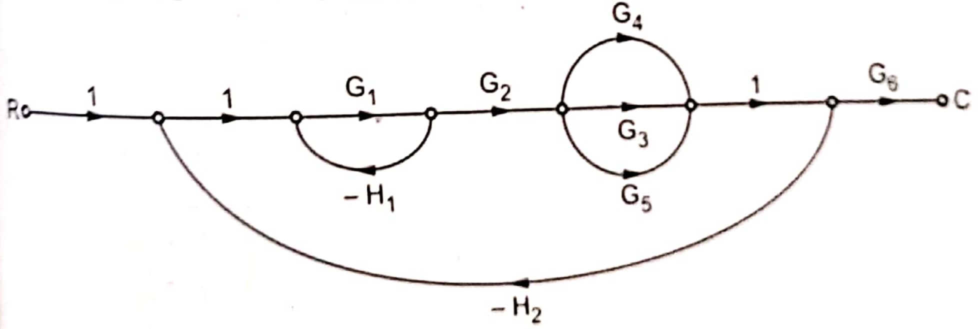


Fig. 1(a)

$$T_1 = G_1G_2G_3G_6, \quad T_2 = G_1G_2G_4G_6, \quad T_3 = G_1G_2G_5G_6$$

Individual loop gains are,

$$L_1 = -G_1H_1, \quad L_2 = -G_1G_2G_3H_2,$$

$$L_3 = -G_1G_2G_4H_2, \quad L_4 = -G_1G_2G_5H_2$$

No combination of nontouching loops.

$$\Delta = 1-[L_1 + L_2 + L_3 + L_4]$$

$\therefore$ All loops are touching to all the forward paths hence $\Delta_1 = \Delta_2 = \Delta_3 = 1$.

By Mason's gain formula,

$$\frac{C}{R} = \frac{T_1\Delta_1 + T_2\Delta_2 + T_3\Delta_3}{\Delta}$$

$$\therefore \quad = \boxed{\frac{G_1G_2G_3G_6 + G_1G_2G_4G_6 + G_1G_2G_5G_6}{1+G_1H_1 + G_1G_2G_3H_2 + G_1G_2G_4H_2 + G_1G_2G_5H_2}}$$

OR

c) *A unity feedback system having open loop transfer function.* $G(s) = \frac{100}{s(s+10)}$. *Determine : i) Error contants* k_p, k_v *and* k_a, *ii) Steady state error for unit step input.* **[4]**

Ans. : Refer Q.8 of Chapter 2 for the procedure and verify answers as : i) $K_p = \infty$, K_v **= 10**, K_a **= 0**, ii) e_{ss} **= 0**

Q.2 a) *Derive an expression for Rise time (Tr) and Settling time (T_s) for second order under damped system for unit step unit.* **[8]**

Ans. : For T_r, refer Q.13 of chapter 2.

Settling time (T_s) : Assuming tolerance limit as ± 2 %,

$$c(t)\big|_{t=T_s} = 0.98.$$

At $t = T_s$, transient term completely vanishes and only term controlling output is $e^{-\xi\omega_n t}$

$$c(t)\big|_{t=T_s} = 1 - e^{-\xi\omega_n T_s} = 0.98$$

$$e^{-\xi\omega_n T_s} = 1 - 0.98 = 0.02$$

Solving,

$$T_s = \frac{3.912}{\xi\omega_n} \approx \boxed{\frac{4}{\xi\omega_n}}$$

b) *Sketch the root locus and common on the stability for a unity feedback system with open loop tranfer function.* **[8]**

$$G(s)H(s) = \frac{K}{s(s+1+j)(s+1-j)}$$

(Refer similar Q.19 of chapter 3)

c) *For a system* $F(s) = s^4 + 22s^3 + 10s^2 + s + K$. *Use Routh's criterion to obtain the marginal value of K and the frequency of oscillations of that value of k.* **[4]**

Ans. : Routh's array is,

$$\begin{array}{c|ccc} s^4 & 1 & 10 & K \\ s^3 & 22 & 1 & 0 \\ s^2 & 9.954 & K & \\ s^1 & \frac{9.954-22K}{9.954} & 0 & \\ s^0 & K & & \end{array}$$

From s^1 row,

$9.954 - 22\,K = 0$

$\therefore K_{mar} = \mathbf{0.4525}$

$A(s) = 9.954\,s^2 + K = 0$

$\therefore s^2 = \frac{-0.4525}{9.954}$

$\therefore s = \pm\, j\, 0.213$

$\therefore$ Frequency of oscillations = **0.213 rad/sec**

Q.3 a) *Compare frequency domain specifications and time domain specifications of control system.* **(Refer Q.4 of chapter 4)** **[8]**

b) *The specifications of standard second order unity feedback control system are that the maximum overshoot must not exceed 30 % and rise time must be less than 0.2 second. Find the limiting values of resonant peak* M_r *and bandwidth.* **[8]**

Ans. : $\% M_p = 30\ \%$, $T_r = 0.2$ sec,

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.3$$

Solving, $\xi = 0.3578$

$$T_r = \frac{\pi-\theta}{\omega_d} \quad \text{and } \theta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi} = 1.205 \text{ rad}$$

$$\therefore \quad 0.2 = \frac{\pi - 1.205}{\omega_d} \quad \text{i.e.} \quad \omega_d = 9.683 \text{ rad/sec}$$

But

$$\omega_d = \omega_n \sqrt{1-\xi^2} \quad \text{i.e.} \quad \omega_n = 10.369 \text{ rad/sec}$$

$$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}} = \mathbf{1.496}$$

$$BW = \omega_n \sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}} = \mathbf{14.55}$$

OR

Q.4 a) *Derive an expression for resonant frequency (ω_r) for standard second order system.* **(Refer Q.2 of chapter 4)** **[8]**

b) *Sketch the polar plot and determine gain margin for a unity feedback system having open loop transfer function* **[8]**

$$G(s) = \frac{1}{s(s+1)(2s+1)}$$ **(Refer similar Q.4 of chapter 6)**

Q.5 a) *State and explain Nyquist stability criterion.*
(Refer Q.8 of chapter 6) **[6]**

b) *The open loop transfer function of a unity feedback system is given by* $G(s) = \frac{10}{s(1+0.5s)(1+0.1s)}$. *Draw Bode plot and determine GM, PM, gain cross over frequency and phase cross over frequency. Comment on stability of the system.* **(Refer similar Q.10 of chapter 5)** **[12]**

OR

Q.6 a) *Define the terms : Cut - off frequency (ω_C), Resonance Peak frequency (M_P), Resonant frequncy (ω_r)*
(Refer Q.6 of chapter 4 and Q.4 of chapter 5) **[6]**

b) *Draw Bode plot for a system having* $G(s)H(s) = \frac{100}{s(s+1)(s+2)}$.

Comment on stability of the system.
(Refer similar Q.7 of chapter 5) **[12]**

Q.7 a) *Draw block diagram and discuss PD controller.*
(Refer Q.5 of chapter 7) **[8]**

b) *Write short note on Synchros.* **(Refer Q.22 of chapter 7)** [8]

OR

Q.8 a) *Differentiate between phase lead and phase lag compensation.* [8]

Ans. : Refer Q.17 of chapter 7 for lag and lead networks and their ransfer functions and pole-zero plots.

Effects of lead compensator : The various effects of a lead compensation are,

1. The lead compensator adds a dominant zero and a pole. This increases the damping of the closed loop system.
2. The increased damping means less overshoot, less rise time and less settling time. Thus there is improvement in the transient response.
3. It improves the phase margin of the closed loop system.
4. The slope of the magnitude plot in Bode diagram of the forward path transfer function is reduced at the gain cross over frequency. This improves gain and phase margins improving the relative stability.
5. It increases bandwidth of the closed loop system. More the bandwidth, faster is the response.
6. The steady state error does not get affected.

Effects of lag compensator : The various effects and limitations of lag compensator are,

1. Lag compensator allows high gain at low frequencies thus it is basically a low pass filter. Hence it improves the steady state performance.
2. In lag compensation, the attenuation characteristics is used for the compensation. The phase lag characteristics is of no use in the compensation.
3. The attenuation due to lag compensator shifts the gain crossover frequency to a lower frequency point. Thus the bandwidth of the system gets reduced.
4. Reduced bandwidth means slower response. Thus rise time and settling time are usually longer. The transient response lasts for longer time.
5. The system becomes more sensitive to the parameter variations.

6. Lag compensator approximately acts as proportional plus integral controller and thus tends to make system less stable.

b) *The system given below is so design to have damping ratio 0.707. Determine the required value of K_p for given damping ratio.* **(Refer similar Q.8 of chapter 7)** [8]

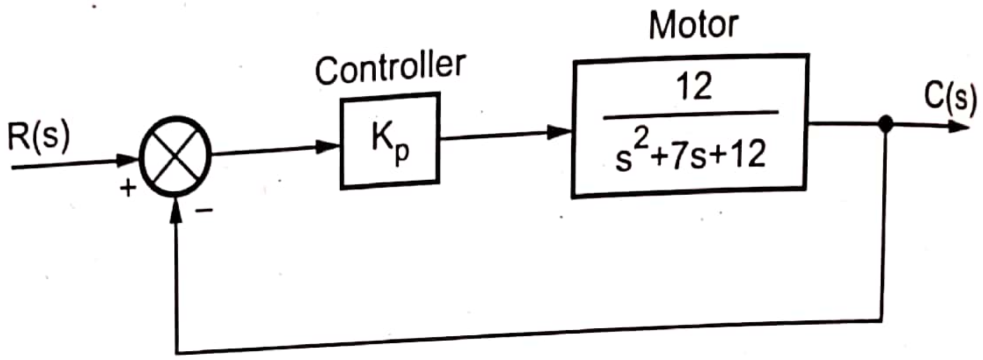


Fig. 2

END...

APRIL - 2019 [IN SEM] [115]

Solved Paper

Course 2015

e : 1 Hour] **[Maximum Marks : 30**

a) *Compare open loop system and close loop system.*

efer Q.5 of chapter 1)

[4]

b) *Find transfer function of given system using block diagram uction.*

[6]

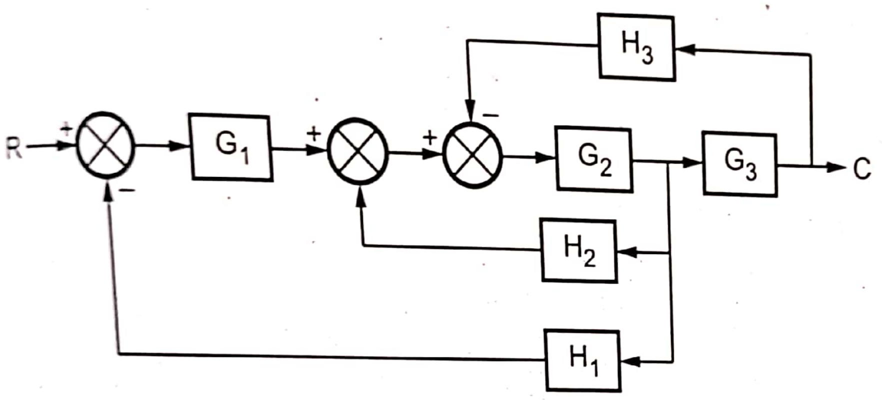
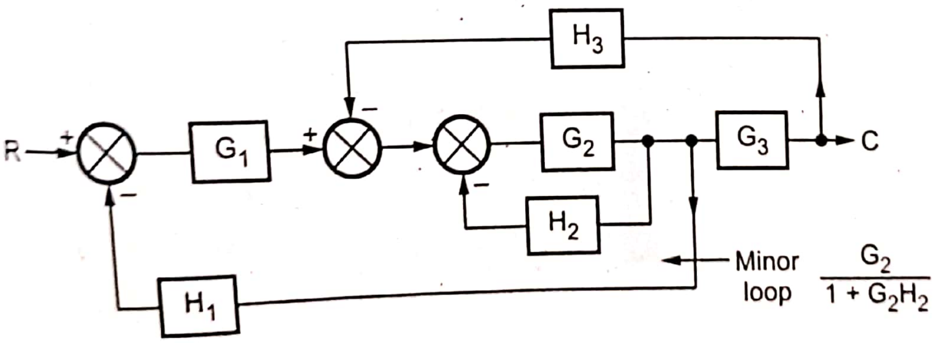
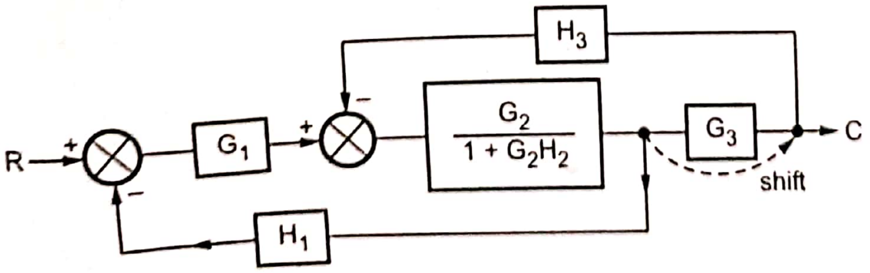
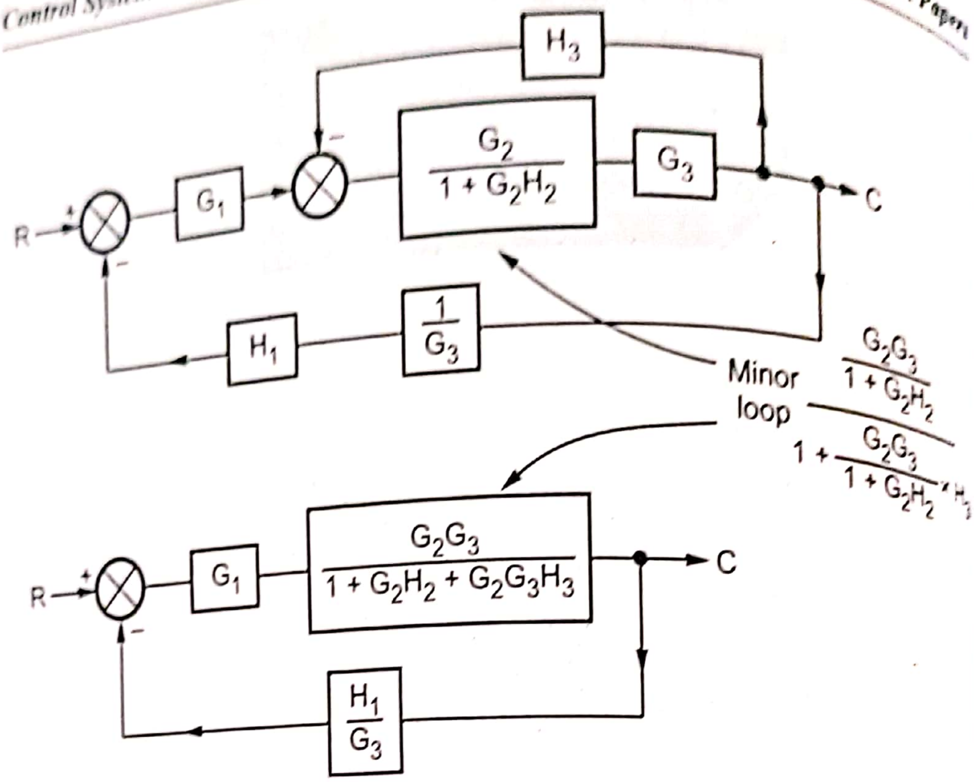

H3
R
+
−
G1
+
+
−
G2
G3
C
H2
H1

Fig. 1

ns. : Change the positions of summing points between G_1 and $_2$.


H3
−
R
+
−
G1
+
G2
G3
C
−
H2
H1
Minor loop
G2 / 1 + G2H2


H3
−
R
+
−
G1
+
G2 / 1 + G2H2
G3
C
shift
H1


H3
G2 / 1 + G2H2
G3
R
G1
C
H1
1 / G3
Minor loop
G2G3 / 1 + G2H2 over 1 + G2G3 / 1 + G2H2 × H3
R
G1
G2G3 / 1 + G2H2 + G2G3H3
C
H1 / G3

$$\therefore \quad \frac{C}{R} = \frac{\dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3}}{1+\dfrac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3}\times\dfrac{H_1}{G_3}}$$

$$\therefore \quad \boxed{\frac{C}{R} = \frac{G_1G_2G_3}{1+G_2H_2+G_2G_3H_3+G_1G_2H_1}}$$

OR

Q.2 **a)** *Explain masons gain formula.* **(Refer Q.33 of chapter 1)** [4]

b) *Find transfer function $V_o(s)/V_i(s)$ for given electrical network.* [6]

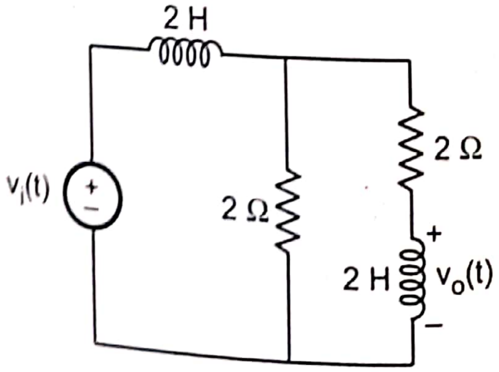

2 H
2 Ω
2 Ω
$v_i(t)$
+
−
+
2 H
$v_o(t)$
−

Fig. 2

∴ The s-domain network is shown in the Fig. 2 (a).

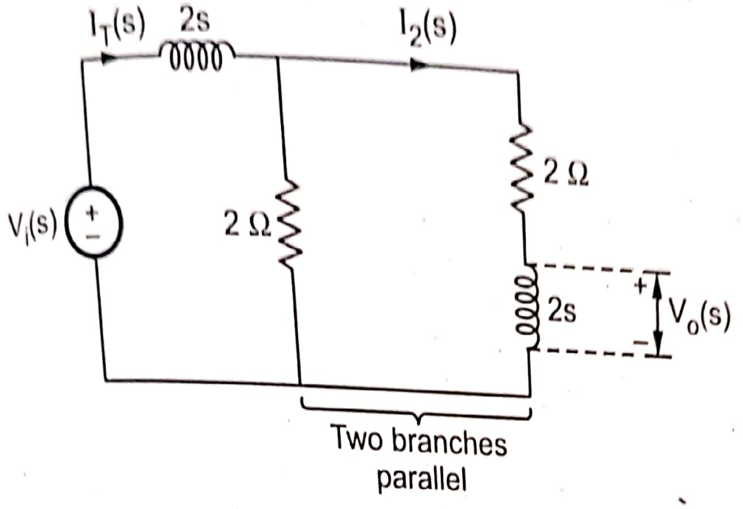
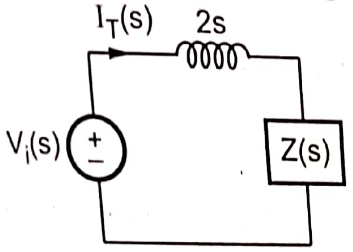


Fig. 2 (a)



$$= 2 \,||\, (2+2s) = \frac{2\times(2+2s)}{2+2+2s}$$

$$= \frac{4\,(s+1)}{2\,(s+2)} = \frac{2\,(s+1)}{(s+2)}$$

Fig. 2 (b)

$$\therefore \quad I_T(s) = \frac{V_i(s)}{2s+\dfrac{2(s+1)}{(s+2)}} = \frac{(s+2)V_i(s)}{(2s^2+6s+2)}$$

Using current division rule to the Fig. 2 (a).

$$I_2(s) = \frac{2\times I_T(s)}{(2+2+2s)} = \frac{2\times(s+2)V_i(s)}{2(s+2)(2s^2+6s+2)} = \frac{V_i(s)}{2s^2+6s+2}$$

$$\therefore \quad V_o(s) = 2s\times I_2(s) = \frac{2sV_i(s)}{2s^2+6s+2}$$

$$\therefore \quad \boxed{\frac{V_o(s)}{V_i(s)} = \frac{s}{2s^2+6s+2}}$$

Q.3 **a)** *Define static error coefficients.* **(Refer Q.3 of chapter 2)** **[4]**

b) *A unity feedback system has the following forward transfer function :* $G(s) = \dfrac{1000(s+8)}{(s+7)(s+9)}$ **[6]**

i) Evaluate system type, K_p, K_v and K_a.

ii) Steady state error for step input and ramp input.

Ans. : i) No pole at the origin in G(s) H(s) hence type of the system is **zero.**

$$K_p = \lim_{s\to 0} G(s)H(s) = \frac{1000\times 8}{7\times 9} = 126.984$$

$$K_v = \lim_{s\to 0} G(s)H(s) = 0$$

$$K_a = \lim_{s\to 0} s^2 G(s)H(s) = 0$$

ii) For step input, $e_{ss} = \dfrac{A}{1+K_p} = \dfrac{1}{1+126.984} = \mathbf{0.00781}$

For ramp input, $e_{ss} = \dfrac{A}{K_v} = \infty$

OR

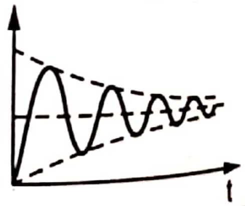
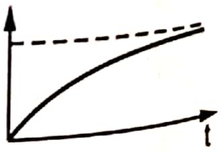
Q.4 a) *For each of the following transfer functions, find its roots and draw, the general form of the step response.* **[6]**

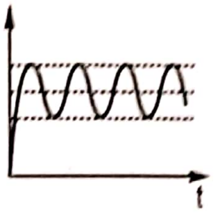
i) $G(s) = \dfrac{400}{s^2+12s+400}$

ii) $G(s) = \dfrac{900}{s^2+90s+900}$

iii) $G(s) = \dfrac{625}{s^2+625}$

Ans. : Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$,

T.F.	ω_n	ξ	**Roots**	**Step response**
$\dfrac{400}{s^2+12s+400}$	20	0.3	$-6 \pm j\,19.078$	
$\dfrac{900}{s^2+90s+900}$	30	1.5	$-11.458,\ -78.542$	

$\frac{625}{s^2+625}$	25	0	± j 25	

b) *For a second order system find ξ, ω_n, T_s, % overshoot.* **[4]**

$$G(s) = \frac{16}{s^2+3s+16}$$

Ans. : Comparing denominator with $s^2+2\xi\omega_n s+\omega_n^2$, $\omega_n^2 = 16$, $\omega_n = 4$ and $2\xi\omega_n = 3$, $\xi = \mathbf{0.375}$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.375\times 4} = \mathbf{2.667\ sec}$$

$$\%\, M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100 = \mathbf{28.06\ \%}$$

Q.5 a) *Given the unity feedback system with* $G(s) = \frac{K(s+4)}{s(s+1.2)(s+2)}$ **[6]**

find the following :

i) The range of K that keeps the system stable

ii) The value of K that makes the system oscillate

. iii) The frequency of oscillation when K is set to

Ans. : The characteristic equation is 1 + G(s)H(s) = 0

$$\therefore \quad 1+\frac{K(s+4)}{s(s+1.2)(s+2)} = 0$$

$$\therefore\ s^3+3.2s^2+s(K+2.4)+4K = 0$$

i) Routh's array :

$$\begin{array}{c|cc} s^3 & 1 & K+2.4 \\ s^2 & 3.2 & 4K \\ s^1 & \frac{3.2K+7.68-4K}{3.2} & 0 \\ & = \frac{7.68-0.8K}{3.2} & \\ s^0 & 4K & \end{array}$$

From s^0, $4K > 0$ $\therefore$ $K > 0$

From s^1, $7.68 - 0.8K > 0$ $\therefore$ $K < \frac{7.68}{0.8} < 9.6$

$\therefore$ Range of K to keep the system stable is **$0 < K < 9.6$.**

ii) From s^1 row, $7.68 - 0.8K = 0$

$\therefore$ $K_{mar} = \mathbf{9.6}$ ...system oscillate

iii) From s^2 row, $A(s) = 3.2s^2 + 4K = 0$

$$\therefore \quad s^2 = \frac{-4K}{3.2} = \frac{-4 \times 9.6}{3.2} = -12$$

$$\therefore \quad s = \pm j\,3.464$$

$\therefore$ Frequency of oscillations = **3.464 rad/sec**

b) *Explain angle and magnitude criterion for Root Locus.*

(Refer Q.13 of chapter 3) [4]

OR

Q.6 a) *Sketch the root locus for the system with open Loop Trans function,* $G(s) = \dfrac{K(s+3)}{s(s+1)(s+2)(s+4)}$ [8]

Find break away point, intersection with imaginary axis, K marginal.

Ans. : Step 1 : P = 4, Z = 1, N = P = 4, P – Z = 3 → ∞

Starting : s = 0, – 1, – 2, – 4

Terminating : s = – 3, ∞, ∞, ∞

Step 2 : Sections of real axis as shown in the Fig. 3 (a).

Step 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z}, \; q = 0, 1, 2$$

$$\therefore \; \theta_1 = 60°, \; \theta_2 = 180°, \; \theta_3 = 300°$$

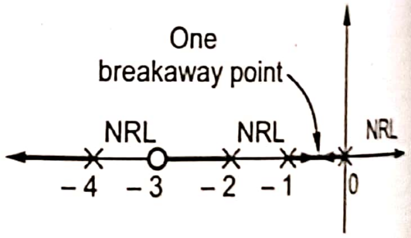


Fig. 3 (a)

Step 4 : Centroid $= \dfrac{\sum \text{R.P. of O.L. poles} - \sum \text{R.P. of O.L. zeros}}{P-Z}$

$$= \frac{0-1-2-4-(-3)}{3} = \mathbf{-1.333}$$

Step 5 : Breakaway point

$$1+G(s)H(s)=0 \quad \text{i.e. } 1+\frac{K(s+3)}{s(s+1)(s+2)(s+4)}=0$$

$$s^4+7s^3+14s^2+8s+K(s+3)=0 \qquad \ldots(1)$$

$$\therefore \qquad K=\frac{-s^4-7s^3-14s^2-8s}{(s+3)} \qquad \ldots(2)$$

$$\frac{dK}{ds}=0 \text{ gives } s^4+8.67s^3+25.66s^2+28s+8=0$$

Solving,

$$s=-0.434,\ -1.617,\ -3.3\pm j\,0.653$$

$$\therefore \qquad s=\mathbf{-0.434} \text{ is valid breakaway point.}$$

Step 6 : Intersection with jω axis

From equation (1), characteristic equation is,

$$s^4+7s^3+14s^2+s(K+8)+3K=0$$

$$\begin{array}{c|ccc}
s^4 & 1 & 14 & 3K \\
s^3 & 7 & K+8 & 0 \\
s^2 & \frac{90-K}{7} & 3K & 0 \\
s^1 & \dfrac{\left(\frac{90-K}{7}\right)(K+8)-21K}{\left(\frac{90-K}{7}\right)} & 0 & \\
s^0 & 3K & &
\end{array}$$

$$\text{From } s^1,\ \left(\frac{90-K}{7}\right)(K+8)-21K=0$$

$$\therefore \qquad K^2+65K-720=0$$

$$\therefore \quad (K-9.645)(K+74.645)=0$$

$$\therefore \qquad K_{mar}=\mathbf{9.645}$$

$$\text{From } s^2 \text{ row,} \qquad A(s)=\left(\frac{90-K}{7}\right)s^2+3K=0$$

$$\therefore \qquad s^2=\frac{-3K}{\left(\frac{90-K}{7}\right)}=-2.52 \text{ i.e., } s=\pm j\,\mathbf{1.588}$$

Step 7 : ϕ_d not required.

Step 8 : The root locus is shown in the Fig. 3 (b).

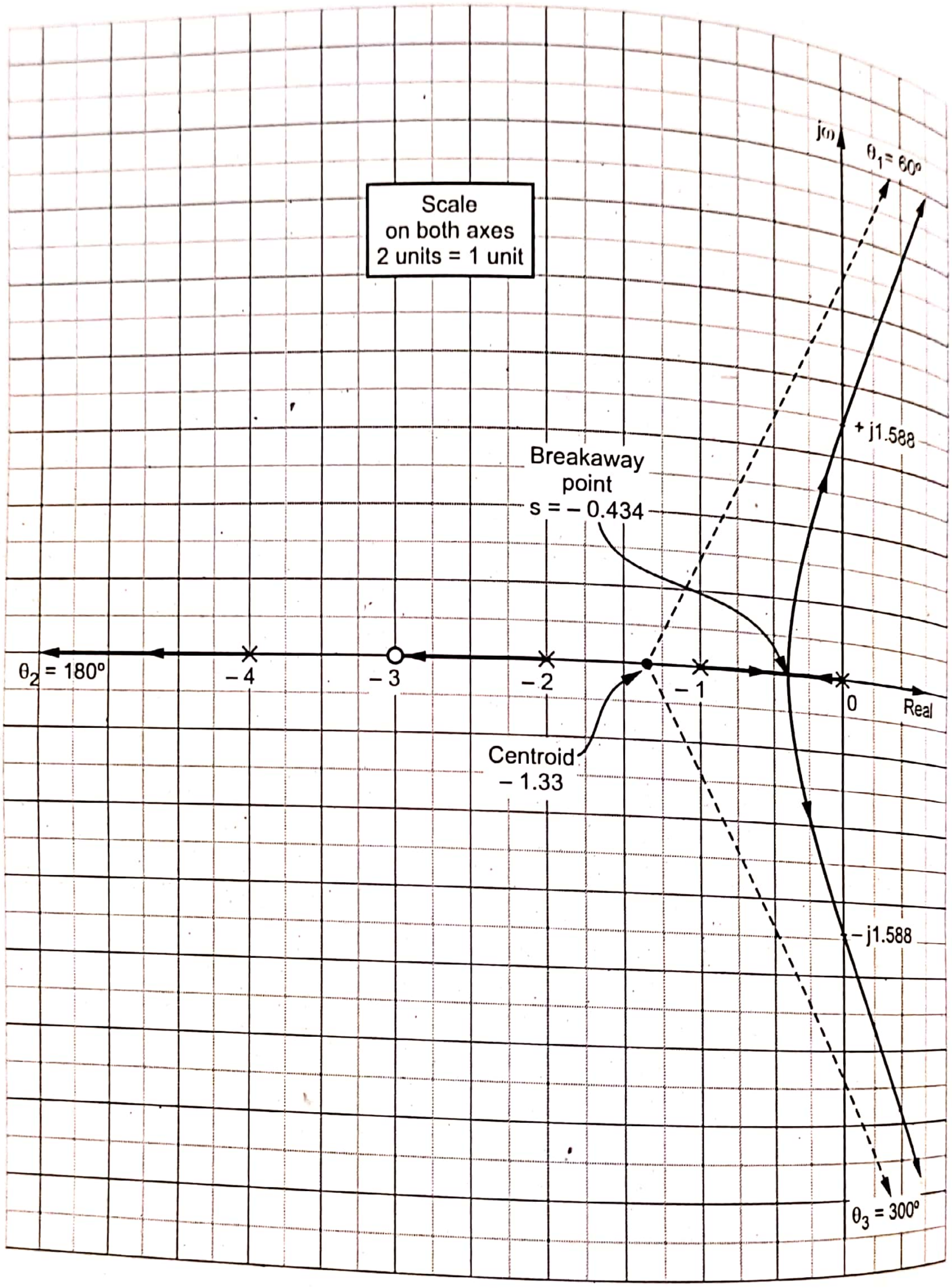


Fig. 3 (b)

b) *What is root locus.* **(Refer Q.13 of chapter 3)** [2]

AY - 2019 [END SEM] [5560] - 566 Solved Paper

Course 2015

me : $2\frac{1}{2}$ Hours] [Maximum Marks : 70

structions to the candidates :

i) *Answer any one question from each pair of questions : Q.1 or Q.2, Q.3 or Q.4, Q.5 or Q.6, Q.7 or Q.8.*

ii) *Figures to the right indicate full marks.*

.1 a) *Draw the electrical analogous network and write the equation.*
Refer Q.19 of Chapter - 1) [7]

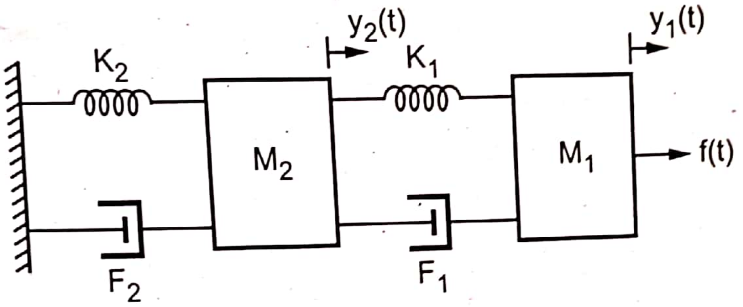


Fig. 1

b) *The poles of a real rational transfer function are given as 0, – 1 and – 4. There is a single zero (or order 2)at s = (– 3). Determine the transfer function and plot pole zero on s-plane.* [5]

Ans. : The transfer function is,

$$T(s) = \frac{(s+3)^2}{s(s+1)(s+4)}$$

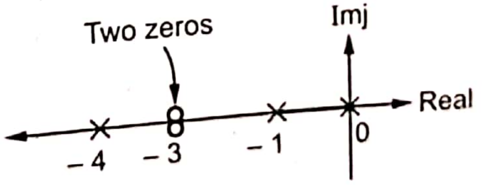


Fig. 2

The pole-zero plot is shown in the Fig. 2.

c) *The block diagram of a unity feedback control system shown in figure below.* [8]

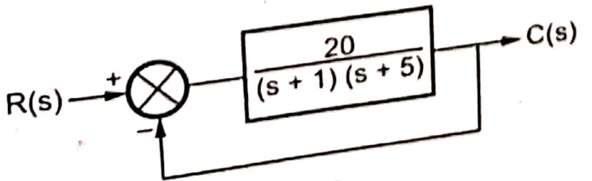


Fig. 3

Determine the characteristic equation of the system ω_n, ξ, ω_d, t_p, M_p *the time at which the first overshoot occurs, the time period of oscillation.* **(Refer Q.19 of Chapter - 2)**

OR

Q.2 a) *Define the following :* **[7]**

i) Time response

(Refer Important point to remember on Page 2-1)

ii) Transient response

(Refer Important point to remember on Page 2-1)

iii) Steady state response

(Refer Important point to remember on Page 2-1)

iv) Delay time **(Refer Q.12 of Chapter - 2)**

v) Rise time **(Refer Q.12 of Chapter - 2)**

vi) Peak time **(Refer Q.12 of Chapter - 2)**

vii) Settling time **(Refer Q.12 of Chapter - 2)**

b) *A characteristic equation of a feedback control system is given by* $s^2 + s^4 + 4s^3 + 4s^2 + 2s + 1 = 0$ *comment on stability.* **[4]**

Ans. : Routh's array is,

$$
\begin{array}{l|lll l}
s^5 & 1 & 4 & 2 & \\
s^4 & 1 & 4 & 1 & \\
s^3 & \boxed{0} + \varepsilon & 1 & 0 & \Leftarrow \text{special case 1} \\
s^2 & \dfrac{4\varepsilon - 1}{\varepsilon} & 1 & & \lim\limits_{\varepsilon \to 0} \dfrac{4\varepsilon - 1}{\varepsilon} \longrightarrow -\text{ve} \\
s^1 & \dfrac{\dfrac{4\varepsilon - 1}{\varepsilon} - \varepsilon}{\left(\dfrac{4\varepsilon - 1}{\varepsilon}\right)} & 0 & & \lim\limits_{\varepsilon \to 0} \dfrac{\dfrac{4\varepsilon - 1}{\varepsilon} - \varepsilon}{\dfrac{4\varepsilon - 1}{\varepsilon}} \longrightarrow +\text{ve} \\
s^0 & 1 & & &
\end{array}
$$

There are two sign changes in the first column of the Routh's array. Hence the **system is unstable.**

c) *A unity feedback control system has an open loop transfer* $G(s) = \dfrac{K}{s(s^2+4s+13)}$ *sketch the root locus of the system by* *rmining the following*

i) centroid and angle of asymptotes

ii) Angle of departure from the poles

iii) The value of K and the frequency at which the root locus *sses the imaginary axis.* **(Refer Q.19 of Chapter - 3)** **[9]**

3 **a)** *Define and write formula* **[8]**

i) Resonant frequency

ii) Resonant peak

iii) Band width

iv) Plot M_r, M_p *versus* ξ *for a second order system*

Refer Q.4 and Q.5 of Chapter - 4)

b) *A unity feedback system has open loop transfer function* **[6]**

$G(s) = \dfrac{(s+2)}{(s+1)(s-1)}$ *using Nyquist criterion determine whether the closed loop system is stable or not.* **(Refer Q.18 of Chapter - 6)**

OR

Q.4 a) *Briefly state the Nyquist criterion.*

(Refer Q.8 of Chapter - 6) **[6]**

b) *Sketch the Bode plot for the system whose open loop transfer function is given by* $G(s) = \dfrac{20(0.1s+1)}{s(0.5s+1)(0.3s+1)}$ *and find GM, PM,* ω_{gc}, ω_{pc}. **[10]**

Ans. : Step 1 : G(s) is in time constant form.

Step 2 : Factors

i) K = 20, 20 log K = 26.02 dB, straight line parallel to log ω axis

ii) $\frac{1}{s}$, one pole at origin, straight line of slope – 20 dB/dec passing through intersection of ω = 1 and 0 dB.

iii) $\frac{1}{(1+0.55)}$, simple pole, $T_1 = 0.5$, $\omega_{c1} = \frac{1}{T_1} = 2$, straight line of slope – 20 dB/dec for $\omega \geq 2$

iv) $\frac{1}{1+0.35}$, simple pole, $T_2 = 0.3$, $\omega_{c2} = \frac{1}{T_2} = 3.33$, straight line of slope – 20 dB/dec for $\omega \geq 3.33$

v) $1 + 0.15$, simple zero, $T_3 = 0.1$, $\omega_{c3} = \frac{1}{T_3} = 10$, straight line of slope + 20 dB/dec for $\omega \geq 10$

Range of ω	$0 < \omega < 2$	$2 \leq \omega < 3.33$	$3.33 \leq \omega < 10$	$10 \leq \omega < \infty$
Resultant slope in dB/dec	– 20	– 20 – 20 = – 40	– 40 – 20 = – 60	– 60 + 20 = – 40

Step 3 : Phase angle table : $G(j\omega) = \frac{20(1+0.1j\omega)}{j\omega\,(1+0.5\,j\omega)(1+0.3\,j\omega)}$

ω	$\frac{1}{j\omega}$	$-\tan^{-1}0.5\omega$	$-\tan^{-1}0.3\omega$	$+\tan^{-1}0.1\omega$	ϕ_R
0.2	– 90°	– 5.71°	– 3.48°	+ 1.145	– 98°
1	– 90°	– 26.56°	– 16.7°	+ 5.71°	– 127.5°
4	– 90°	– 63.48°	– 50.19°	+ 21.8°	– 181.8°
10	– 90°	– 78.69°	– 71.56°	+ 45°	– 195.25°
∞	– 90°	– 90°	– 90°	+ 90°	– 180°

Step 4 : The bode plot is shown in the Fig. 4. The answers are also given in the Fig. 4. (See Fig. 4 on next page)

Q.5 a) *Define gain margin, phase margin, phase crossover frequncy, gain crossover frequency.* **(Refer Q.4 of Chapter - 5)** **[6]**

b) *Sketch the asymptotic plot for open loop transfer function given by* $G(s) = \frac{2(s+0.25)}{s^2(s+1)(s+0.5)}$ *from Bode diagram determine GM, PM,* ω_{gc}*,* ω_{pc}. **[12]**

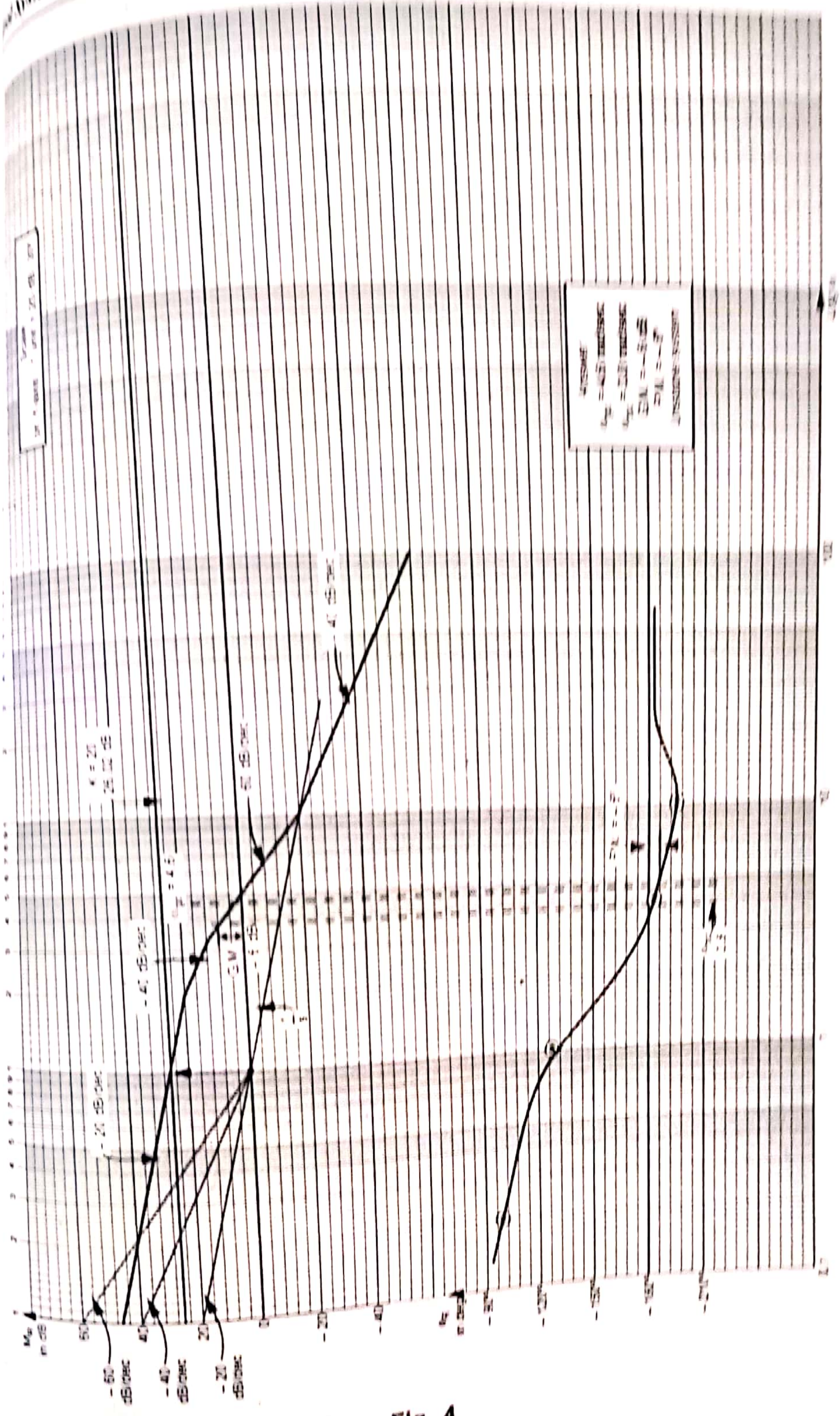

M in dB
60
40
20
0
-20
-40
- 60 dB/dec
- 40 dB/dec
- 20 dB/dec
- 20 dB/dec
- 40 dB/dec
- 60 dB/dec
- 40 dB/dec

Fig. 4

Ans. : **Step 1 :** G(s) is in time constant form.

$$G(s) = \frac{(1+4s)}{s^2(1+s)(1+2s)}$$

Step 2 : Factors

i) K= 1, 20 log K = 0 dB, no contribution by constant.

ii) $\frac{1}{s^2}$, 2 poles at origin, straight line of slope – 40 dB/dec passing through intersection of $\omega = 1$ and 0 dB.

iii) 1 + 45, simple zero, $T_1 = 4$, $\omega_{C1} = \frac{1}{T_1} = 0.25$, straight line of slope + 20 dB/dec for $\omega \geq 0.25$

iv) $\frac{1}{1+2s}$, simple pole, $T_2 = 2$, $\omega_{C2} = \frac{1}{T_2} = 0.5$, straight line of slope – 20 dB/dec for $\omega \geq 0.5$

v) $\frac{1}{1+s}$, simple zero, $T_3 = 1$, $\omega_{C3} = \frac{1}{T_3} = 1$, straight line of slope – 20 dB/dec for $\omega \geq 1$.

Range of ω	$0 < \omega < 0.25$	$0.25 \leq \omega < 0.5$	$0.5 \leq \omega < 1$	$1 \leq \omega < \infty$
Resultant slope in dB/dec	– 40	– 20 + 20 = – 20	– 20 – 20 = – 40	– 40 – 20 = – 60

Step 3 : Phase angle table : $G(j\omega) = \frac{1+4j\omega}{(j\omega)^2(1+j\omega)(1+2j\omega)}$

ω	$\frac{1}{(j\omega)^2}$	$+\tan^{-1}4\omega$	$-\tan^{-1}\omega$	$-\tan^{-1}2\omega$	ϕ_R
0.2	– 180°	+ 38.66°	– 11.3°	– 21.8°	– 174.44°
0.5	– 180°	+ 63.43°	– 26.56°	– 45°	– 188.13°
1	– 180°	+ 75.96°	– 45°	– 63.43°	– 212.48°
∞	– 180°	+ 90°	– 90°	– 90°	– 270°

Step 4 : The bode plot and the answers are given in the Fig. 5.

b) *Using Nyquist criterion investigate the stability of a closed loop control system whose open loop transfer function is given by* **[12]**

$$G(s) = \frac{K}{s\,(sT_1+1)(sT_2+1)}$$

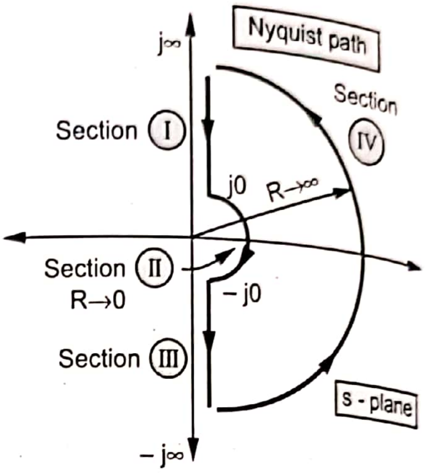


Fig. 6 (a)

Ans. :

Step 1 : P = 0, no open loop pole in R.h.s of s-plane.

Step 2 : For stability, N = – P = 0

Step 3 : The Nyquist path is as shown in the Fig. 6 (a).

Step 4 : $G(j\omega) = \dfrac{K}{(j\omega)(1+j\omega T_1)(1+j\omega T_2)}$

Section I : s = j∞ to s = + j0

Starting ω → ∞	$0\angle\frac{0^\circ}{90^\circ 90^\circ 90^\circ} = 0\angle{-270^\circ}$	– 90° – (– 270°) = + 180° Anticlockwise
Terminating ω → + 0	$\infty\angle\frac{0^\circ}{40^\circ 0^\circ 0^\circ} = \infty\angle{-90^\circ}$	

Section II : s = + j0 to s = – j0

Starting ω → +0	$\infty\angle{-90^\circ}$	– 90° – (– 90°) = + 180° Anticlockwise
Terminating ω → –0	$\infty\angle{+90^\circ}$	

Section III : Mirror image of section I about real axis.

Section IV : Not required for closed loop stability.

Step 5 : Intersection with real axis

Rationalize (jω),

$$G(j\omega) = \frac{-j\omega(1-j\omega T_1)(1-j\omega T_2)K}{-j\omega(-j\omega)(1+j\omega T_1)(1-j\omega T_1)(1+j\omega T_2)(1-j\omega T_2)}$$

$$= \frac{[-\omega^2(T_1+T_2)-j\omega(1-\omega^2 T_1 T_2)K}{\omega^2(1+\omega^2 T_1^2)(1+\omega^2 T_2^2)}$$

imaginary part to be zero, $-\omega^2 T_1 T_2 = 0$

$$\omega_{pc} = \frac{1}{\sqrt{T_1 T_2}}$$

bstituting in real part, the inter section point with negative real xis is,

$$Q = \frac{-(T_1+T_2)K}{\left(1+\frac{T_1^2}{T_1T_2}\right)\left(1+\frac{T_2^2}{T_1T_2}\right)}$$

$$= \frac{-T_1T_2K}{T_1+T_2}$$

Step 6 : The Nyquist plot is shown in the Fig. 6 (b).

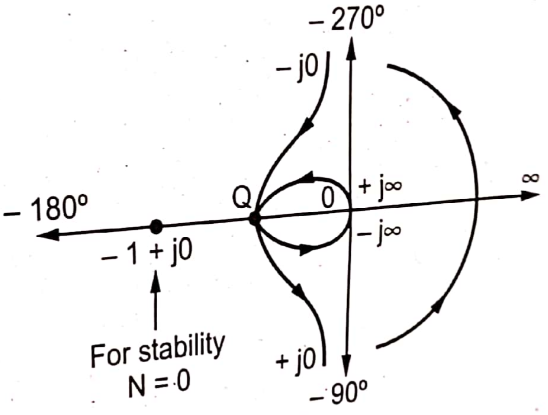


Fig. 6 (b)

Step 7 : For stability, N = 0 i.e. $|OQ| < 1$

$$\therefore \frac{T_1T_2K}{T_1+T_2} < 1 \quad \text{i.e.} \quad K < \frac{T_1+T_2}{T_1T_2} < \frac{1}{T_1} + \frac{1}{T_2}$$

[8]

Q.7 a) *Write short note on*

i) Lead compensator **(Refer Q.17 of Chapter - 7)**

ii) AC Tachometer

Ans. : ii) The a.c. tachometer is a device which is very much similar to a two phase induction motor.

In this type two stator windings are placed in quadrature with each other and rotor is short circuited.

The schematic diagram is as shown in Fig. 7.

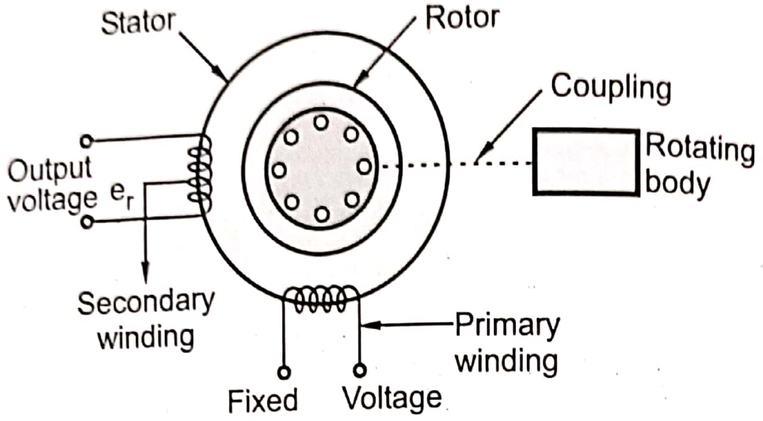


Fig. 7

For an a.c. tachometer, a sinusoidal voltage of rated value is applied to the primary winding which is also known as reference winding.

The secondary winding is placed at a 90° mechanically apart in space from the primary winding.

When rotor shaft is rotated, the magnitude of the sinusoidal output voltage e_T will be proportional to rotor speed. Thus, when the rotor shaft is stationary, the output voltage is zero. The polarity of the output voltage is determined by the direction of rotation.

The transfer function of an a.c. tachometer is,

$$e(t) = K\frac{d\theta}{dt} \qquad \text{i.e.} \qquad \boxed{\frac{E(s)}{\theta(s)} = K_s}$$

where E(s) = Laplace transform of the output voltage.

θ (s) = Laplace transform of the rotor position.

K = Constant

b) *Explain the features of the following* **[8]**

i) P-controller **(Refer Q.1 of Chapter - 7)**

ii) PI-controller **(Refer Q.4 of Chapter - 7)**

iii) PID-controller **(Refer Q.6 of Chapter - 7)**

OR

Q.8 **a)** *Write short notes on synchros.* **(Refer Q.22 of Chapter - 7)[6]**

b) *For the system shown below, design PID controller using Zigler Nichol tuning rule.* **[10]**

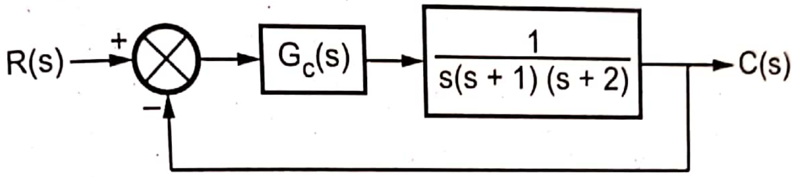


Fig. 8

Ans. : Let K_p is proportional controller gain.

$$\therefore \quad G(s) = K_p G(s) = \frac{K_p}{s(s+1)(s+2)}$$

The characteristic equation is $1 + G(s)H(s) = 0$

$$\therefore 1+\frac{K_p}{s(s+1)(s+2)} = 0 \quad \text{i.e.} \quad s^3+3s^2+2s+K_p = 0$$

Routh's array is,

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 3 & K_p \\ s^1 & \frac{6-K_p}{3} & 0 \\ s^0 & K_p & \end{array}$$

For critical value of K_p

$$6-K_p = 0$$

$$\therefore K_p = \mathbf{K_u} = 6$$

$$A(s) = 3s^2+K_p = 0$$

$$\therefore s^2 = \frac{-K_p}{3} = \frac{-6}{3} = -2$$

$$\therefore s = \pm j\,1.414 = \pm j\omega$$

$$\omega = 1.414 = 2\pi f = \frac{2\pi}{T_u} \qquad \ldots f = \frac{1}{T_u}$$

$$\therefore \quad T_u = \frac{2\pi}{\omega} = \frac{2\pi}{1.414} = \mathbf{4.443}\ \text{sec}$$

$\therefore$ According to Ziegler-Nichols method,

$K_p = 0.6\,K_u = \mathbf{3.6}$, $T_i = 0.5\,T_u = \mathbf{2.2215}$, $K_d = \frac{T_u}{8} = \mathbf{0.555}$

Hence the T.F of PID controller is,

$$G_c(s) = \frac{0.075\,K_u\,T_u\left(s+\frac{4}{T_u}\right)^2}{s} = \frac{1.9935(s+0.9)^2}{s}$$

$$\therefore \quad G_{eq}(s) = \frac{1.9935(s+0.9)^2}{s^2(s+1)(s+2)}$$

DEC. - 2019 [END SEM] [5669]-537 Solved Paper

Course 2015

Time : $2\frac{1}{2}$ Hours] **[Maximum Marks : 70**

Q.1 a) *Find transfer function using Mason gain formula.* **[7]**

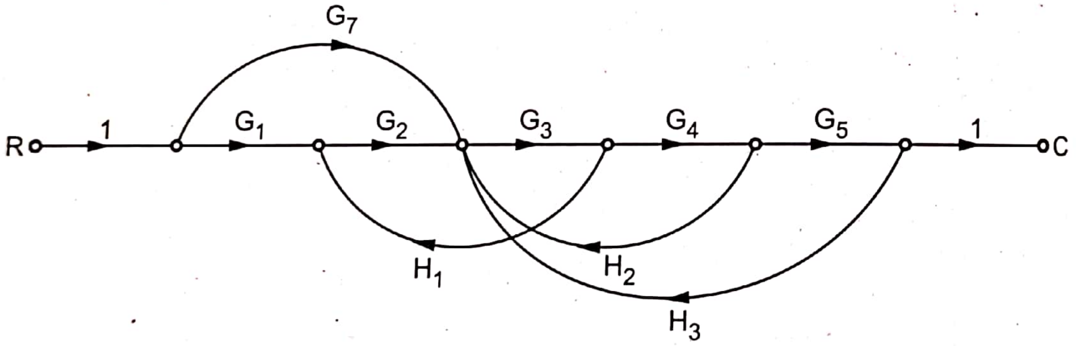
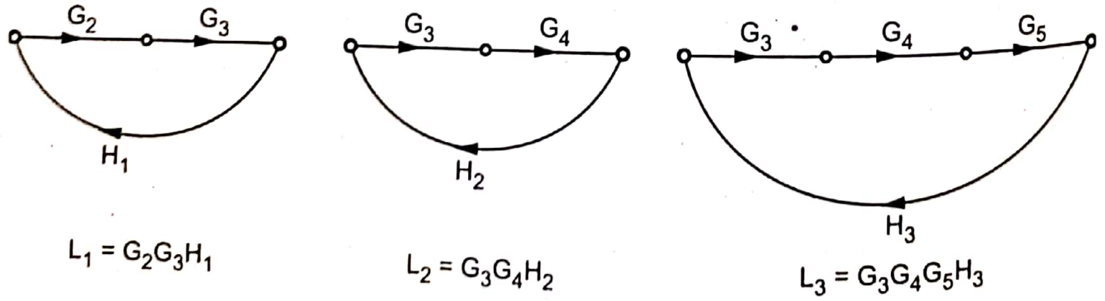


Fig. 1

Ans. : Forword path gains are,

$$T_1 = G_1G_2G_3G_4G_5, \quad T_2 = G_7G_3G_4G_5$$

The individual feedback loop gains are,



$L_1 = G_2G_3H_1$ $L_2 = G_3G_4H_2$ $L_3 = G_3G_4G_5H_3$

Fig. 1 (a)

No combination of non touching loops.

$$\therefore \quad \Delta = 1-(L_1+L_2+L_3) = 1-G_2G_3H_1-G_3G_4H_2-G_3G_4G_5H_3$$

For both T_1 and T_2, as loops all touching hence $\Delta_1 = \Delta_2 = 1$.

$$\therefore \quad \frac{C}{R} = \frac{T_1\Delta_1+T_2\Delta_2}{\Delta} = \boxed{\frac{G_1G_2\,G_3\,G_4G_5\;+G_7G_3\,G_4G_5}{1-G_2G_3\,H_1-G_3G_4H_2-G_3G_4G_5H_3}}$$

b) *For the unity feedback system with* $G(s) = \dfrac{4900}{s(s+70)}$ *find peak overshoot, settling time for units step input. What is steady error for an input of 5tu(t).* **[6]**

Ans. : The characteristic equation is $1+G(s)H(s) = 0$.

$$\therefore \quad 1+\frac{4900}{s(s+70)} = 0 \text{ i.e. } s^2+70s+4900 = 0 \text{ i.e.} s^2+2\xi\omega_n s+\omega_n^2 = 0$$

$$\therefore \quad \omega_n^2 = 4900,\ \omega_n = 70 \text{ and } 2\xi\omega_n = 70,\ \xi = 0.5$$

$$\%\,M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}\times 100 = \mathbf{16.303\ \%}$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.5\times 70} = \mathbf{0.1143\ sec}$$

$$\text{Input } r(t) = 5t\ u(t),\ A = 5$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0}\frac{s\times 4900}{s(s+70)} = 70$$

$$\therefore \quad e_{ss} = \frac{A}{K_v} = \frac{5}{70} = \mathbf{0.0714}$$

c) *Construct Root Locus for unity feedback system with open loop transfer function given as* $G(s) = \dfrac{K(s+1.5)}{s(s+1)(s+2)}$. **[7]**

Ans. : Step 1 : P = 3, Z = 1, N = P = 3, Z – P = 2 →∞,

Starting : s = 0, –1, –2, Terminating : s = 1.5, ∞, ∞

Step 2 : Sections of real axis as shown in the Fig. 2 (a)

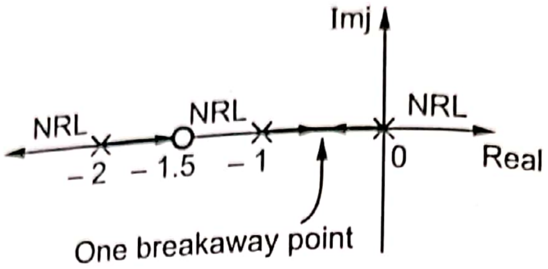


Fig. 2 (a)

Step 3 : Angles of asymptotes

$$\theta = \frac{(2q+1)180°}{P-Z}, \quad q = 0, 1$$

$$\therefore \quad \theta_1 = 90°, \quad \theta_2 = 270°$$

Step 4 : Centroid $= \dfrac{\sum \text{R.P. of O.L. poles} - \sum \text{R.P. of O.L. Zeros}}{P-Z}$

$$= \frac{0-1-2-(-1.5)}{2} = -0.75$$

Step 5 : Breakway point

$$1 + G(s)H(s) = 0 \text{ gives } 1 + \frac{K(s+1.5)}{s(s+1)(s+2)} = 0$$

$$\therefore \quad s^3 + 3s^2 + 2s + K(s+1.5) = 0 \qquad \text{...(1)}$$

$$\therefore \quad K = \frac{-s^3 - 3s^2 - 2s}{(s+1.5)} \qquad \text{...(2)}$$

$$\therefore \quad \frac{dK}{dS} = 0$$

gives $(s + 1.5)(-3s^2 - 6s - 2) - (-s^3 - 3s^2 - 2s)(1) = 0$

$$\therefore \quad 2s^3 + 7.5s^2 + 9s + 3 = 0$$

Solving $s = -0.544, \ -1.603 \pm j\,0.4311$

From step 2, $S = -$ **0.544** is valid breakaway point and

from eqaution (2), $K = +$ **0.378** at this breakaway point.

Step 6 : Intersection with jω axis

From equation (1), the characteristic equation is

$$s^3 + 3s^2 + s(2 + K) + 1.5K = 0$$

s^3	1	$2 + K$	from row of s^1
s^2	3	1.5 K	$6 + 1.5\ K = 0$ $\therefore K_{mar} = -4$
s^1	$\frac{6+1.5K}{3}$	0	As K_{mar} is negtive, the root locus does not intersect imaginary axis and lies completely in left half of s-plane.
s^0	1.5 K		

Step 7 : Angle of departure not required.

Step 8 : The root locus is shown in Fig. 2 (b).

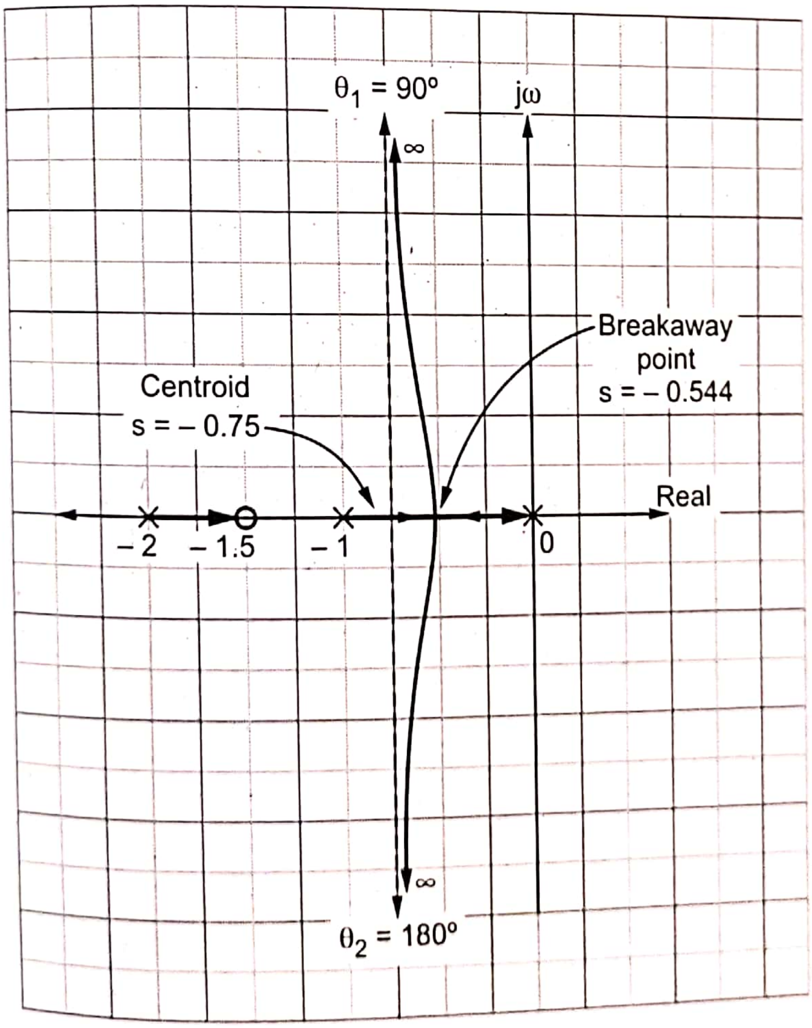

θ₁ = 90°
jω
∞
Breakaway point s = – 0.544
Centroid s = – 0.75
Real
– 2
– 1.5
– 1
0
∞
θ₂ = 180°

Fig. 2 (b)

OR

Q.2 a) *Draw electrical analogues network for given mechanical system in F-V analogy and write differential equations.* **[7]**

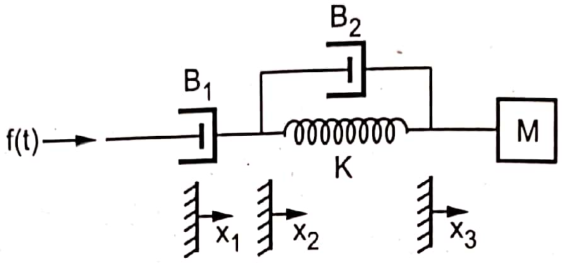


Fig. 3

Ans. : There are 3 displacements x_1, x_2 and x_3. Elements B_1 is between x_1 and x_2, K and B_2 are between x_2 and x_3. The mass M is under x_3. The equivalent mechanical system is shown in the Fig. 3 (a).

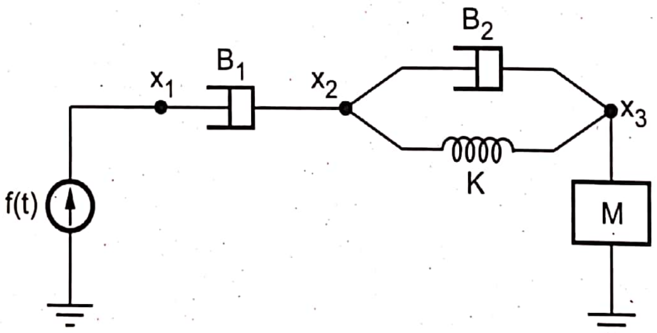


Fig. 3 (a)

The equations are

$$f(t) = B_1 \frac{d(x_1 - x_2)}{dt},$$

$$0 = B_1 \frac{d(x_2 - x_1)}{dt} + B_2 \frac{d(x_2 - x_3)}{dt} + K(x_2 - x_3)$$

$$0 = B_2 \frac{d(x_3 - x_2)}{dt} + K(x_3 - x_2) + M\frac{d^2 x_3}{dt^2}$$

For F-V analogy use : $M \to L, B \to R, K - \frac{1}{C}, x \to q, \dot{x} \to \frac{dq}{dt} \to i,$

$x \to \int i \, dt, \ \ddot{x} = \frac{di}{dt}$

$$v = R_1(i_1 - i_2)$$

$$0 = R_1(i_2 - i_1) + R_2 + (i_2 - i_3) + \frac{1}{C}\int (i_2 - i_3)\,dt$$

$$0 = R_2(i_3 - i_2) + \frac{1}{C}\int (i_3 - i_2)\,dt + L\frac{di_3}{dt}$$

The F-V analogous electrical network is shown in the Fig. 3 (b).

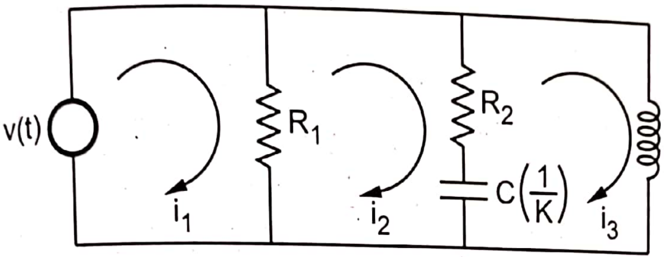


Fig. 3 (b)

b) *For system with* $G(s) = \frac{K}{s(Ts+1)}$, *H(s) = 1 is subjected to a unit step input, the system output responds as shown in Fig. 4. Determine the values of K and T from response curve.* **[6]**

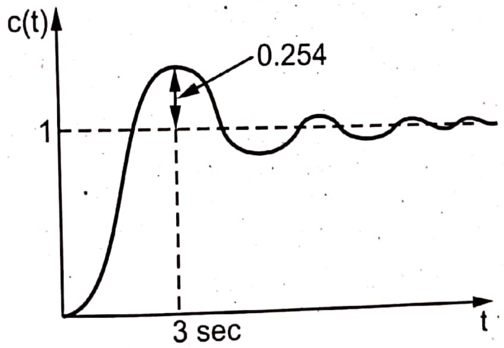


Fig. 4

Ans. : From Fig. 4, the overshoot is 0.254 i.e. 25.4 %.

$\therefore$ $$0.254 = e^{-\pi\xi/\sqrt{1-\xi^2}} \quad \text{i.e. } ln(0.254) = \frac{-\pi\xi}{\sqrt{1-\xi^2}}$$

Solving, $\xi = \mathbf{0.4}$

$T_p = 3$ sec from Fig. 4 i.e. $T_p = \frac{\pi}{\omega_d} = 3$

$\therefore$ $$\omega_d = 1.0472 = \omega_n\sqrt{1-\xi^2} \quad \text{and } \xi = 0.4$$

$$\therefore \quad \omega_n = \mathbf{1.1426\ rad/sec}$$

The characteristics equation is 1 + G(s)H(s) = 0.

$$\therefore \quad 1+\frac{K}{s(1+Ts)} = 0 \text{ i.e. } Ts^2+s+K=0$$

$$\text{i.e. } s^2+\frac{1}{T}s+\frac{K}{T}=0$$

Compare with $s^2+2\xi\omega_n s+\omega_n^2=0$

$$\therefore \quad \omega_n^2 = \frac{K}{T},\ \omega_n = \sqrt{\frac{K}{T}} \text{ and } 2\xi\omega_n = \frac{1}{T},\ \xi=\frac{1}{2\sqrt{KT}}.$$

$$\therefore \quad 0.4 = \frac{1}{2\sqrt{KT}} \text{ i.e. } KT = 1.5625 \qquad \text{...(1)}$$

$$\therefore \quad 1.1426 = \sqrt{\frac{K}{T}} \text{ i.e. } 1.3055 = \frac{K}{T} \qquad \text{...(2)}$$

Solving, K = **1.4282**, T = **1.094**

c) *Find the positive value of K and a such that system with unity feedback and* $G(s)=\dfrac{K(s+1)}{(s^3+as^2+2s+1)}$ *oscoillates at frequency of 2 rad/sec.* **[7]**

Ans. : The characteristic equation is 1 + G(s)H(s) = 0

$$\therefore \quad 1+\frac{K(s+1)}{s^3+as^2+2s+1} = 0$$

$$\text{i.e. } s^3+as^2+s(2+K)+(1+K) = 0$$

Routh's array,

s^3	1	2 + K
s^2	a	1 + K
s^1	$\dfrac{a(2+K)-(1+K)}{a}$	0
s^0	1+ K	

For K_{mar}

$$a(2+K)-(1+K) = 0$$

$$\therefore \ a = \frac{K+1}{K+2} \qquad \text{... (1)}$$

$$A(s) = as^2+(1+K) = 0$$

$$\therefore \ s^2 = \frac{-(1+K)}{a}$$

$$s = \pm j\sqrt{\frac{1+K}{a}} = \pm j\omega \quad \ldots\ \omega = 2 \text{ rad/sec given}$$

$$\sqrt{\frac{1+K}{a}} = 2 \text{ i.e. } \frac{K+1}{a} = 4 \text{ i.e. } a = \frac{K+1}{4} \qquad \ldots(2)$$

Equating (1) and (2), $\frac{K+1}{K+2} = \frac{K+1}{4}$ i.e. $K + 2 = 4$

$$K = \mathbf{2} \text{ and } a = \frac{3}{4} = \mathbf{0.75}$$

Q3 **a)** *Explain correlation between frequency domain and time domain.* (Refer Q.4 of Chapter 4) **[6]**

b) *Sketch polar plot for system with open loop transfer function as* $G(s) = \frac{50}{(s+3)(s+6)}$. *Obtain gain margin and phase margin.* **[10]**

Ans. : Refer Q.4 of Chapter 6.

$$G.M = 20 \text{ Log } \frac{1}{|OQ|} = 20 \text{ Log} \frac{1}{0.308} = \mathbf{10.23\ dB}$$

$$G(j\omega) = \frac{50}{j\omega(j + j\omega)(6 + j\omega)} \text{ i.e. } |G(j\omega)| = \frac{50}{\omega\sqrt{9+\omega^2}\sqrt{36+\omega^2}}$$

At ω_{gc}, $|G(j\omega)| = 1$ i.e. $1 = \frac{50}{\omega\sqrt{9+\omega^2}\sqrt{36+\omega^2}}$

$\therefore\ \omega^2(9+\omega^2)(36+\omega^2) = 50^2$

$\therefore\ \omega^6 + 45\omega^4 + 324\omega^2 - 2500 = 0$

Solving, $\omega_{gc} = 2.133$ rad/sec

$$\therefore\ P.M. = 190° + \angle G(j\omega)\Big|_{\omega=\omega_{gc}}$$

$$= 180° + \frac{0°}{90°, \tan^{-1}\frac{\omega}{3}, \tan^{-1}\frac{\omega}{6}}\Bigg|_{\omega=2.133}$$

$$= \mathbf{+\ 35°}$$

OR

Q.4 a) *Explain Nyquist stability criterion.*

(Refer Q.8 of Chapter 6) [6]

b) *For Unity feedback system* $G(s) = \dfrac{K}{(s+2)(s+4)(s+6)}$ *plot Nyquist diagram. Find range of K for stability.* [10]

Ans. : Step 1 : P = 0, no open loop pole on right hand side of s-plane.

Step 2 : N = –P = 0 for stability

Step 3 : The Nyquist path, is shown in the Fig. 5 (a).

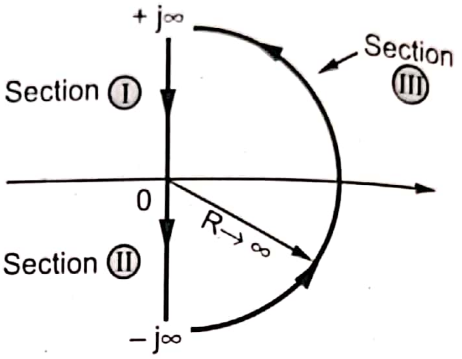


Fig. 5 (a)

Step 4 : $$G(j\omega)H(j\omega) = \frac{K}{(2+j\omega)(4+j\omega)(6+j\omega)}$$

Section I : s = j∞ to s = 0

Starting $\omega \to \infty$	$0\angle -270^\circ$	$0^\circ-(-270^\circ)=+270^\circ$ Anticlockwise
Terminating $\omega = 0$	$\frac{K}{48}\angle 0^\circ$	

Section II : Mirror image of section I about real axis.

Section III : Not required for closed loop stability.

Step 5 : Intersection with real axis

Rotionalize G(jω)H(jω) and equate imaginary part to zero

$$G(j\omega)H(j\omega) = \frac{K(2-j\omega)(4-j\omega)(6-j\omega)}{(2+j\omega)(2-j\omega)(4+j\omega)(4-j\omega)(6+j\omega)(6-j\omega)}$$

$$= \frac{\{(48-12\,\omega^2)+j\omega(\omega^2-44)\}K}{(4+\omega^2)(16+\omega^2)(36+\omega^2)}$$

$$\therefore \quad \omega^2 - 44 = 0$$

i.e. $$\omega_{pc} = \sqrt{44} = 6.633 \text{ rad/sec}$$

$$Q = \frac{(48-12\times 44)K}{(4+44)(16+44)(36+44)} = \frac{-480\,K}{230400} = \frac{-K}{480}$$

tep 6 : The Nyquist plot is shown in the Fig. 5 (b).

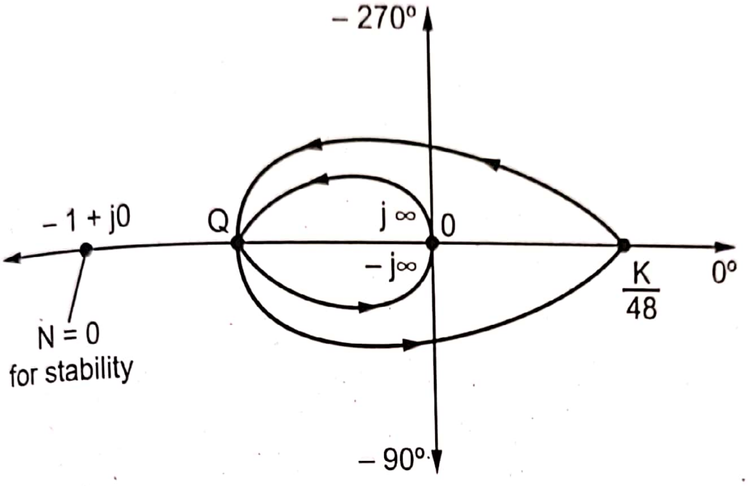


Fig. 5 (b)

Step 7 : For stability, point Q must be to the right of –1 + j0.

$\therefore$ |0Q| < 1 i.e. $\left|\frac{-K}{480}\right| < 1$

$\therefore$ K < 48°

Hence the range of K for stability is $\boxed{0 < K < 48°}$

Q.5 a) *Explain terms gain cross over frequency, phase cross over frequency, gain margin and phase margin.* **(Refer Q.4 of Chapter 5)** **[6]**

b) *Sketch the Bode plot for system with open loop transfer function as G(s) =* $\frac{75(1+0.2s)}{s(s+3)}$. *Determine from that* ω_{gc} ω_{pc}, *GM, PM and comment on stability.* **[12]**

Ans. : Step 1 : G(s) in time contant form.

$$G(s) = \frac{25(1+0.2\,s)}{s[1+0.333\,s)}$$

Step 2 : Factors

i) K = 25, 20 Log K = 27.95 dB, straight line parallel to logω axis

ii) $\frac{1}{s}$, one pole at origin, straight line of slope –20 dB/dec passing through intersection of ω = 1 and 0 dB.

iii) $\frac{1}{(1+0.333\,s)}$, simple pole, $T_1 = 0.333$. $\omega_{C1} = \frac{1}{T_1} = 3$, Straight line of slop −20 dB/dec for $\omega \geq 3$.

iv) $(1+0.2\ s)$ simple zero, $T_2 = 0.2$, $\omega_{C2} = \frac{1}{T^2} = 5$, Straight line of slop +20dB/dec for $\omega \geq 5$.

Range of ω	$0 < \omega < 3$	$3 \leq \omega < 5$	$5 \leq \omega < \infty$
Resultant slope in dB/sec	−20	− 20 − 20 = −40	− 40 + 20 = − 20

Step 3 : Phase angle table : $G(j\omega) = \dfrac{25(1+0.2\,j\omega)}{j\omega(1+0.333\,j\omega)}$

ω	$\frac{1}{j\omega}$	$-\tan^{-1} 0.333\omega$	$+\tan^{-1} 0.2\omega$	ϕ_R
0.3	− 90°	− 5.71°	+ 3.433°	−92.27°
3	− 90°	− 45°	+ 30.96°	−104.03°
10	− 90°	− 73.3°	+63.43°	− 99.86°
100	− 90°	−88.28°	+ 87.13°	− 91.14°
∞	− 90°	−90°	+90°	− 90°

Step 4 : The Bode plot and answers are given in the Fig. 6.
(See Fig. 6 on next page)

OR

Q.6 a) *State advantages of Bode plot.* **(Refer Q.13 of Chapter 5)** **[6]**

b) *Sketch Bode plot for system with open loop transfer function as*

$$G(s) = \frac{K}{s\,(1+0.02s)(1+0.04s)}$$

consider K = 1. Determine from that ω_{gc}, ω_{pc}, *GM, PM.* **[12]**

Ans. : Refer Q.7 of Chapter 5 for the procedure and verify the answers for K =1 as, ω_{gc} = 1 rad/sec., ω_{pc} = 35.35 rad/sec., G.M. = +32 dB, P.M. = +86°, Stable system.

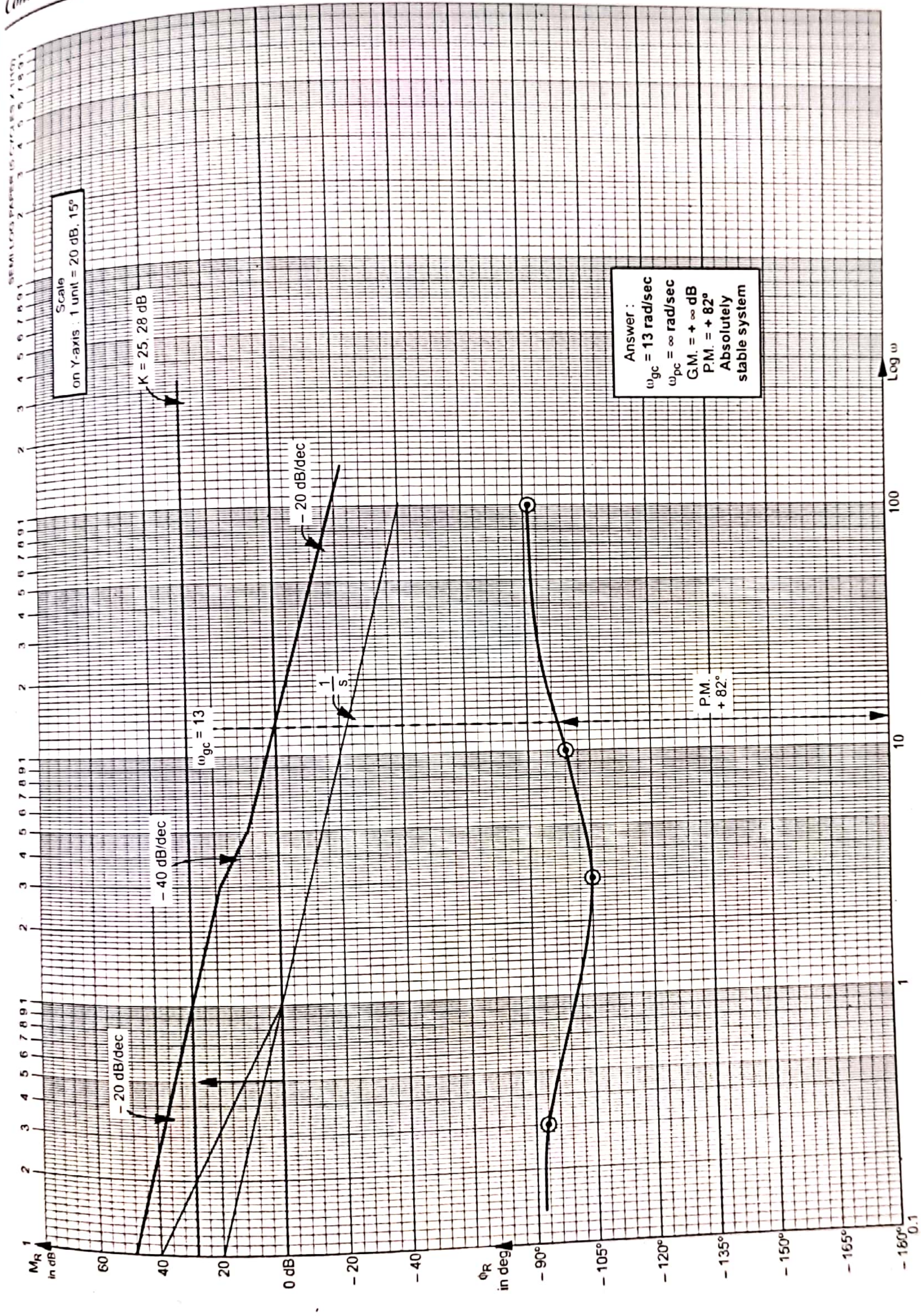


Fig. 6

Q.7 **a)** *Explain P, PI, PID controller with their advantages and limitations.* **(Refer Q.1, Q.4 and Q.6 of Chapter 7)** **[8]**

b) *Obtain the tuning of PID controller for a unity feedback system with open loop transfer function as using Ziegler Nichols method*

$$G(s) = \frac{12}{s(s^2+4s+13)}.$$

[8]

Ans. : Let K is the proportional controller gain.

$$\therefore \quad G(s) = \frac{12\,K}{s(s^2+4s+13)}, \; H(s) = 1$$

The characteristic equation is 1+ G(s) H(s) = 0

$$\therefore \quad 1+\frac{12\,K}{s(s^2+4s+13)} = 0 \text{ i.e. } s^2+4s^2+13s+12\,K = 0$$

Routh's array is

$$\begin{array}{c|cc} s^3 & 1 & 13 \\ s^2 & 4 & 12\,K \\ s^1 & \frac{s2-12K}{4} & 0 \\ s^0 & 12\,K & \end{array}$$

From s^1 row,

$52-12\,K = 0$ For K_{mar}

$\therefore \; K_{mar} = K_u = \mathbf{4.333}$

$A(s) = 4s^2+12K = 0$

$\therefore \; s^2 = -13 \qquad \ldots K = 4.333$

$$\therefore \quad s = \pm j\,3.605 = \pm j\omega \quad \text{i.e. } \omega = 3.605$$

$$\omega = 2\pi f = \frac{2\pi}{T_u} = 3.605 \text{ i.e. } T_u = \mathbf{1.743}$$

According to Ziegler - Nichols methods

$$K_p = 0.6\,K_u = \mathbf{2.6}$$

$$T_i = 0.5\,T_u = \mathbf{0.8715},$$

$$K_d = \frac{T_u}{8} = \mathbf{0.2178}$$

$$\therefore \quad G_c(s) = 0.75\,\frac{K_u T_u \left(s+\frac{4}{T_u}\right)^2}{s}$$

$$\boxed{= \frac{5.664\,(s+2.295)^2}{s}}$$

OR

Q.8 a) *Explain Ziegler Nichols method of PID tuning when dynamic model of system is not available.* **[8]**

Ans. : This method of Ziegler-Nichols approach is used when the dynamics of the plant are unknown.

Step 1 : In this method, the open loop response of the plant is obtained experimentally for a step input. It is assumed that the plant does not have integrators or complex conjugate poles hence its unit step response is S-shaped curve as shown in the Fig. 7. If experimental determination of such a curve is not possible then it is obtained by dynamic simulation of the plant.

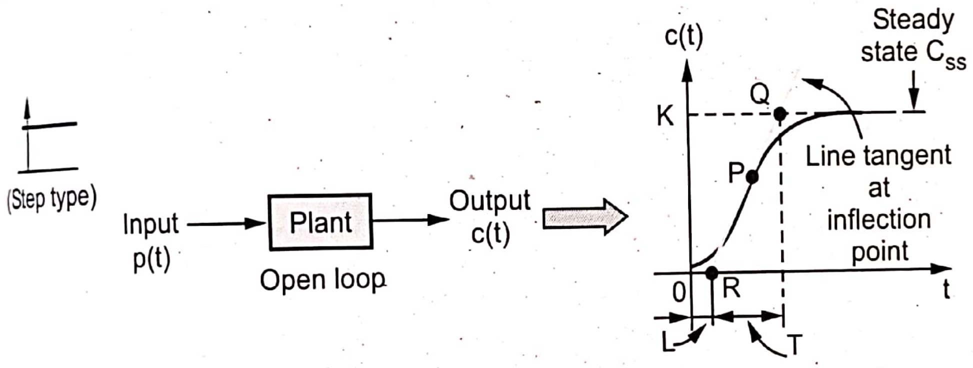


Fig. 7 S-shaped response of the plant

Step 2 : The S-shaped response has two important characteristics called delay time L and time constant T. Draw the tangent line at inflection point P of the S-shaped curve as shown in the Fig. 7. The intersection of this tangent line with steady state line C_{ss}. $C_{ss} = K$ is at point Q while with time axis is at point R. The time corresponding to point R gives the value of L while the time corresponding to the point Q gives the value of T.

Step 3 : Once L and T are known then the transfer function model of the first order system can be modeled with delay time L and time constant T as,

$$\frac{C(s)}{P(s)} = \frac{Ke^{-Ls}}{1+Ts}$$

K is the steady state value of the output which is C_{ss}.

Step 4 : The suggested values of K_p, T_i and K_d according Ziegler-Nichols method for such a case are given in the Table 1.

Controller type	K_p	T_i	K_d
P	$\frac{T}{L}$	∞	0
PI	$\frac{0.9T}{L}$	$\frac{L}{0.3}$	0
PID	$\frac{1.2T}{L}$	2L	0.5 L

Table 1

Thus the transfer function of the controller becomes,

$$G_c(s) = K_p + \frac{K_p}{T_i}\frac{1}{s} + K_p K_d s = \frac{1.2T}{L} + \frac{1.2T}{L(2L)}\frac{1}{s} + \frac{1.2T}{L} \times 0.5\,Ls$$

$$= \frac{1.2T}{L} + \frac{0.6T}{L^2 s} + 0.6Ts = \frac{0.6T}{s}\left[\frac{2s}{L} + \frac{1}{L^2} + s^2\right]$$

$$\boxed{G_c(s) = \frac{0.6T}{s}\left(s + \frac{1}{L}\right)^2}$$

b) *Obtain the tuning of PID controller for a unity feedback system with open loop transfer function as given* $G(s) = \dfrac{1}{(s+1)(s+3)(s+5)}$ *using Ziegler Nichols method.* **[8]**

Ans. : Refer Q.7 (b) for the procedure.

$1 + G(s)H(s) = 0$ gives $s^3 + 9s^2 + 23s + (15 + K) = 0.$

Using Routh's arrary, $K_{mar} = K_u = 192$, $T_u = 1.31$.

$$\therefore \quad K_p = \mathbf{115.2},\ T_i = \mathbf{0.655},\ K_d = \mathbf{0.1637}$$

$$\therefore \quad \boxed{G_c(s) = \frac{188.64 + (s + 3.053)^2}{s}}$$

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ISBN 978-93-332-0092-9

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